

## 1 Advanced Physical Chemistry – Exercises

**1.1** For H<sup>35</sup>Cl the wavenumber of the rotation transition  $J = 0 \rightarrow 1$  is 20.8784 cm<sup>-1</sup>, while it amounts to 10.7840 cm<sup>-1</sup> for D<sup>35</sup>Cl. Do the molecules have the same bond length?

$$M(\text{H}) = 1.007985 \text{ g/mol}, M(\text{D}) = 2.0140 \text{ g/mol}, M(\text{Cl}) = 34.96885 \text{ g/mol}$$

**1.2** The intensity of spectral lines resulting from the transition between vibrational states of a molecule is proportional to the square of the integral  $\int \psi_{v'} x \psi_v \, dx$ . The unnormalized wave functions of an harmonic oscillator having the potential  $V(x) = \frac{1}{2}Kx^2$  are

$$\psi_v = H_v(y) e^{-\frac{y^2}{2}}, \quad y = x/\alpha, \quad \alpha^2 = \frac{\hbar}{\sqrt{\mu K}},$$

where  $H_v(y)$  denotes a Hermitian polynomial. Prove that only transitions with  $v' = v \pm 1$  are allowed!

*Hints:* Hermitian polynomials can be obtained from the recursion relation

$$H_{v+1} = 2yH_v - 2vH_{v-1}.$$

Furthermore, there exists an orthogonality relation,

$$\int_{-\infty}^{\infty} H_{v'} H_v e^{-y^2} \, dy = \begin{cases} 0 & \text{if } v \neq v' \\ \sqrt{\pi} 2^v v! & \text{if } v = v' \end{cases}$$

**1.3** The infrared spectrum of N<sub>2</sub>O consists of 3 fundamental bands.

- Can one deduce from this—assuming that N<sub>2</sub>O is linear—whether the molecule has the structure NNO or NON?
- Sketch the motions of the atoms for the fundamental vibrations of both structures!

**1.4** The infrared spectrum of HCl shows absorption lines at 2885.65, 5667.18, 8344.62, and 10917.94 cm<sup>-1</sup>. Calculate the dissociation energy  $D_0$ , the zero point energy, and the depth of the vibration potential minimum  $D_e$ !

**1.5** The chlorine molecule can be regarded as an anharmonic oscillator having the wavenumber  $\tilde{\nu} = 564.9 \text{ cm}^{-1}$  and the anharmonicity constant  $x_e \tilde{\nu} = 4 \text{ cm}^{-1}$ . Calculate

- the zero-point energy
- the dissociation energy
- the number of vibration levels

Hint: Look for the maximum of the vibration energy.