4 Advanced Physical Chemistry – Exercizes

4.1 How many ways are there to assign positions to the 10 field players of a soccer team?
What do you obtain if you apply Stirling's approximation?

4.2 ... an old safety problem: Should an airplane that is used for crossing the Atlantic Ocean have 1, 2, 3, or 4 motors?

4.3 What is the probability that at least 2 people from a group of 30 celebrate their birthday on the same date?

For simplicity's sake assume that a year has 365 days (ignore switch years) and that there are no seasonal fluctuations of the birth rate.

4.4 Natural carbon consists mostly of the nuclide ^{12}C. The nuclide ^{13}C constitutes merely 1.1%. There are more nuclides, but they can be neglected here.

a) Use the binomial distribution to compute the fractions of molecules in a sample of pentane which contain 0, 1, ... 5 ^{13}C atoms.

b) What is the fraction of pentane molecules in which two ^{13}C atoms occupy neighbour positions?

c) Solve a) using the Poisson distribution. Are there deviations? If yes, why?

4.5 Consider the binomial distribution $P_n(k)$ for very large values of n and k (so that it is permissible to use Stirling's approximation): What is the most probable value of k?
Does the Poisson distribution give the same result?

4.6 The speed distribution function of the molecules of an ideal gas in x direction is

$$f(v_x)\,\mathrm{d}v_x = A\exp\left(-\frac{mv_x^2}{2k_\mathrm{B}T}\right)\,\mathrm{d}v_x\ ,$$

where m denotes the molecular mass and k_B Boltzmann's constante. Analoguous expression exist for the y and z directions.

a) Calculate the normalizing factor A!

b) Give expressions for the average speed, the average absolute speed, and the average kinetic energy of the molecules!

useful equations:

$$\int_0^\infty x^k e^{-\lambda x^2}\,\mathrm{d}x = \frac{1}{2}\lambda^{-\frac{k+1}{2}}\Gamma\left(\frac{k+1}{2}\right) \quad \text{for} \quad \lambda > 0, k > -1$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \qquad \Gamma(1) = 1$$

$$\Gamma(x+1) = x\Gamma(x)$$