Concepts of Space and Time in Quantum Gravity

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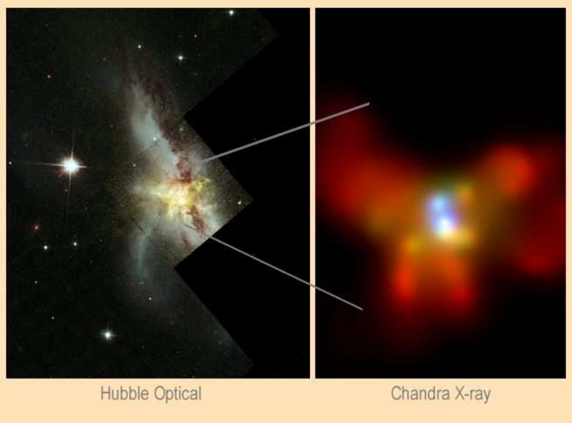
Contents

Why quantum gravity?

- Unification of all interactions
- Singularity theorems
  - Black holes
  - ‘Big bang’
- Problem of time
- Absence of viable alternatives

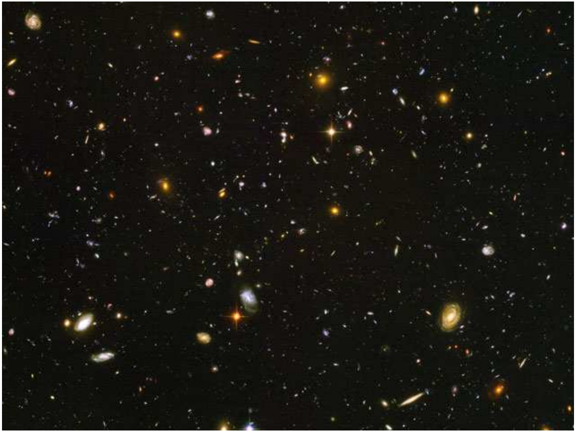
Black holes

The supermassive black holes of the galaxy NGC 6240



View into the early universe

The Universe is about 13,7 billion years old



Wolfgang Pauli (1955):

Es scheint mir . . . , daß nicht so sehr die Linearität oder Nichtlinearität Kern der Sache ist, sondern eben der Umstand, daß hier eine allgemeinere Gruppe als die Lorentzgruppe vorhanden ist . . . .

Matvei Bronstein (1936):

The elimination of the logical inconsistencies connected with this requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we see here, with values unobservable in principle, and perhaps also the rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts. *Wer's nicht glaubt, bezahlt einen Taler*.

The problem of time

- Absolute time in quantum theory:

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

- Dynamical time in general relativity:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

QUANTUM GRAVITY?

Max Planck, Über irreversible Strahlungsvorgänge, *Sitzungsberichte der königlich-preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse*, Seiten 440–80 (1899)

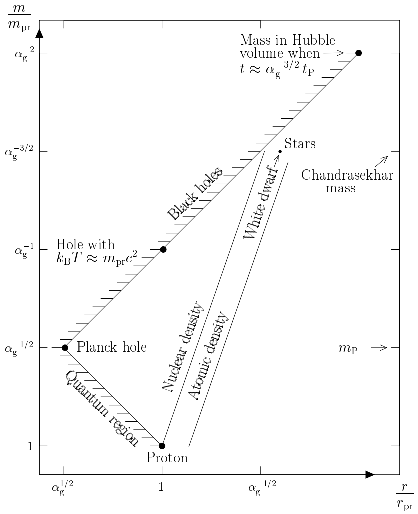
Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ \mathrm{cm} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Structures in the universe



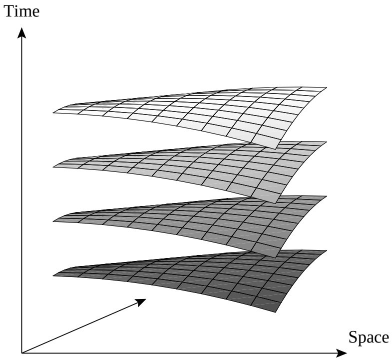
$$\alpha_g = \frac{G m_{pr}^2}{\hbar c} = \left(\frac{m_{pr}}{m_P}\right)^2 \approx 5.91 \times 10^{-39}$$

Main approaches to quantum gravity

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Other approaches
  (Quantization of topology, . . .)

Topic here: canonical quantum geometrodynamics

Canonical formalism



$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt) \\ &= (h_{ab} N^a N^b - N^2) dt^2 + 2 h_{ab} N^a dx^b dt + h_{ab} dx^a dx^b \,. \end{aligned}$$

- configuration variable: three-metric h_{ab}
- canonical momentum π^{ab}: embedding

Constraints

Einstein's equations can be written as a dynamical system of evolution equations together with constraints:

$$\mathcal{H}[h,\pi] \;=\; 2\kappa\, G_{ab\,cd}\pi^{ab}\pi^{cd} \;-\; (2\kappa)^{-1}\sqrt{h}({}^{(3)}R - 2\Lambda) \;+\; \sqrt{h}\rho \approx 0\,,$$
$$D^a[h,\pi] \;=\; -2\nabla_b\pi^{ab} \;+\; \sqrt{h}j^a \approx 0\,,$$

with "DeWitt metric"

$$G_{ab\,cd} = \frac{1}{2\sqrt{h}}(h_{ac}h_{bd} + h_{ad}h_{bc} - h_{ab}h_{cd})$$

$$\kappa = 8\pi G/c^4$$

Full Hamiltonian H is a combination of the constraints!

Configuration space: Space of all three-geometries (=Superspace)

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Distinction according to choice of variables:

- Geometrodynamics –
  variables are three-metric and extrinsic curvature
- Connection dynamics –
  variables are connection (A_a^i) and non-abelian electric field (E_i^a)
- Loop dynamics –
  variables are holonomy connected with A_a^i and the flux of E_i^a

Quantum geometrodynamics

Formal quantization of the classical constraints yields (for the vacuum case):

$$\hat{\mathcal{H}}\Psi \equiv \left(-2\kappa\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (2\kappa)^{-1}\sqrt{h}\big({}^{(3)}R - 2\Lambda\big)\right)\Psi = 0$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraints
(guarantee the coordinate independence of Ψ)

Time, and therefore spacetime, have disappeared from the formalism!

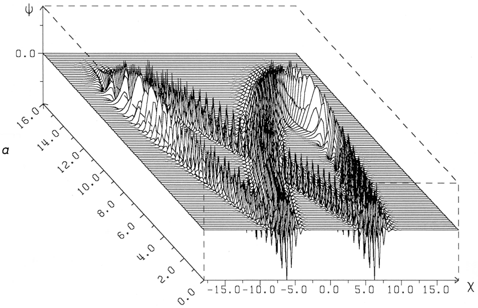
Problem of time

- External time t has vanished from the formalism
- This holds also for loop quantum gravity and probably for string theory
- Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an “intrinsic time”
- Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?
- What is an observable in quantum gravity?

Example

Indefinite oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



(C. K. 1990)

Where does the i in the Schrödinger equation come from?

Erwin Schrödinger 1926:

Eine gewisse Härte liegt ohne Zweifel zurzeit noch in der Verwendung einer *komplexen* Wellenfunktion. Würde sie *grundsätzlich* unvermeidlich und nicht eine bloße Rechenerleichterung sein, so würde das heißen, daß grundsätzlich *zwei* Wellenfunktionen existieren, die erst *zusammen* Aufschluß über den Zustand des Systems geben.

Born–Oppenheimer approximation

Expansion of the Wheeler–De-Witt equation into powers of the Planck mass:

$$\Psi \approx \exp(\mathrm{i}S_0[h]/G\hbar)\,\psi[h,\phi]$$

(h: three-metric, ϕ: non-gravitational degrees of freedom)

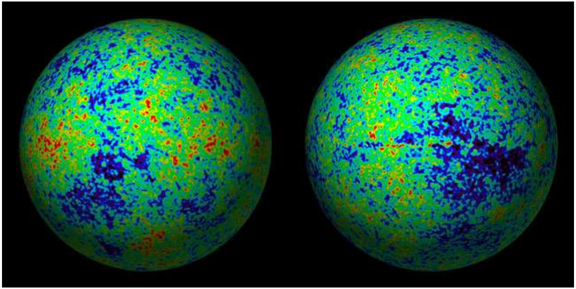
- S_0 obeys the Hamilton–Jacobi equation of gravity
- ψ obeys approximately a (functional) Schrödinger equation:

$$\mathrm{i}\hbar\,\underbrace{\nabla S_0\,\nabla\psi}_{\frac{\partial\psi}{\partial t}} \approx H_{\mathrm{m}}\,\psi$$

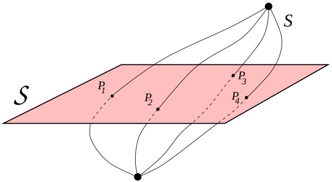
(H_{m}: Hamiltonian for non-gravitational fields ϕ)
Temps t retrouvé!

- Next order: Quantum gravitational corrections to the Schrödinger equation (C. K. and T. P. Singh 1991)

Does the anisotropy spectrum of the Cosmic Background Radiation contain information about quantum gravity?



Loop quantum gravity



Quantization of area:

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_{\mathrm{P}}^2 \sum_{P\in S\cap\mathcal{S}} \sqrt{j_P(j_P+1)}\,\Psi_S[A]$$

(β: free parameter in this approach)

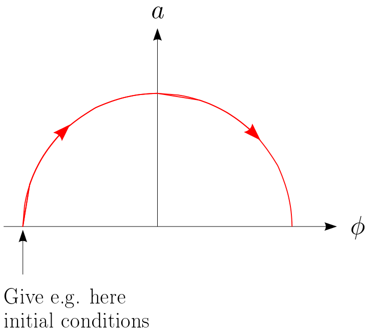
Why quantum cosmology?

Gell-Mann and Hartle 1990:
Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

- Quantum theory is universally valid:
  Application to the Universe as a whole as the only closed quantum system in the strict sense
- Need quantum theory of gravity, since gravity dominates on large scales

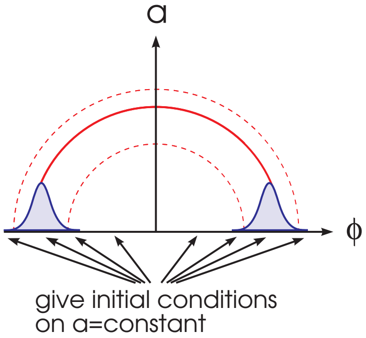
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory

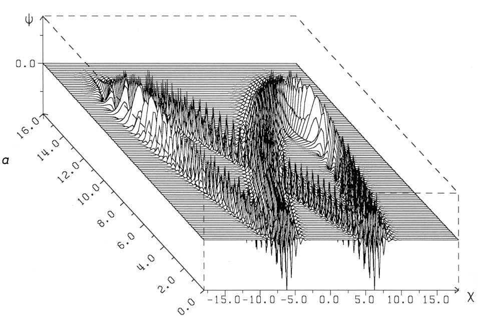


"Recollapsing" wave packet must be present "initially"

Example

Indefinite oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



(C. K. 1990)

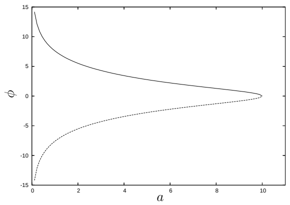
Quantum cosmology with big brake

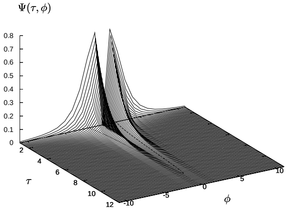
Classical model: Equation of state $p = A/\rho$, $A > 0$, for a Friedmann universe with scale factor $a(t)$ and scalar field $\phi(t)$ with potential ($24\pi G = 1$)

$$V(\phi) = V_0 \left(\sinh(|\phi|) - \frac{1}{\sinh(|\phi|)} \right) ;$$

develops pressure singularity (only $\ddot{a}(t)$ becomes singular)

Quantum model: Normalizable solutions of the Wheeler–DeWitt equation vanish at the classical singularity





(Kamenshchik, C. K., Sandhöfer 2007)

How special is the universe?

Penrose (1981):

Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be

$$\frac{\exp\left(\frac{S}{k_{\mathrm{B}}}\right)}{\exp\left(\frac{S_{\mathrm{max}}}{k_{\mathrm{B}}}\right)} \sim \frac{\exp\left(10^{88}\right)}{\exp\left(10^{123}\right)} \approx \exp\left(-10^{123}\right)$$

Origin of time direction

Fundamental asymmetry with respect to “intrinsic time”:

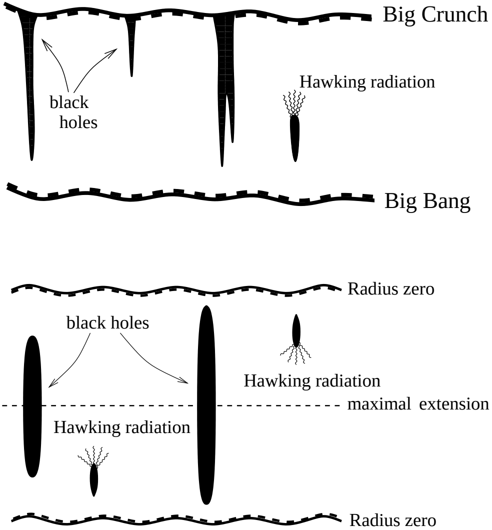
$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial \alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\rightarrow 0 \text{ for } \alpha \rightarrow -\infty} \right] \right) \Psi = 0$$

Is compatible with simple boundary condition:

$$\Psi \overset{\alpha \rightarrow -\infty}{\longrightarrow} \prod_i \psi_i(x_i)$$

Entropy increases with increasing α, since entanglement with other degrees of freedom increases
$\longrightarrow$ defines time direction

Is the expansion of the Universe a tautology?



(C. K. and Zeh 1995)

Loop quantum cosmology

- The Wheeler–DeWitt equation becomes a *difference equation*
- Singularity avoidance: $a = 0$ is not among the discrete steps of this equation
- Inflationary scenario can be accommodated
- but: not yet derived from full loop quantum gravity

Summary

- Time is absent at the most fundamental level;
- time emerges as an approximate concept on the semiclassical level;
- direction of (semiclassical) time can be understood from quantum cosmology;
- space may become discrete near the Planck scale;
- quantum picture from unified theory not yet fully understood.

John Wheeler 1968:

These considerations reveal that the concepts of spacetime and time itself are not primary but secondary ideas in the structure of physical theory. These concepts are valid in the classical approximation. However, they have neither meaning nor application under circumstances when quantum-gemoetrodynamical effects become important. . . . There is no spacetime, there is no time, there is no before, there is no after. The question what happens “next” is without meaning.

Literature: C. K., *Quantum Gravity* (Second edition, Oxford 2007)