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A PTOLEMAIC PAPYRUS FROM THE MICHIGAN COLLECTION

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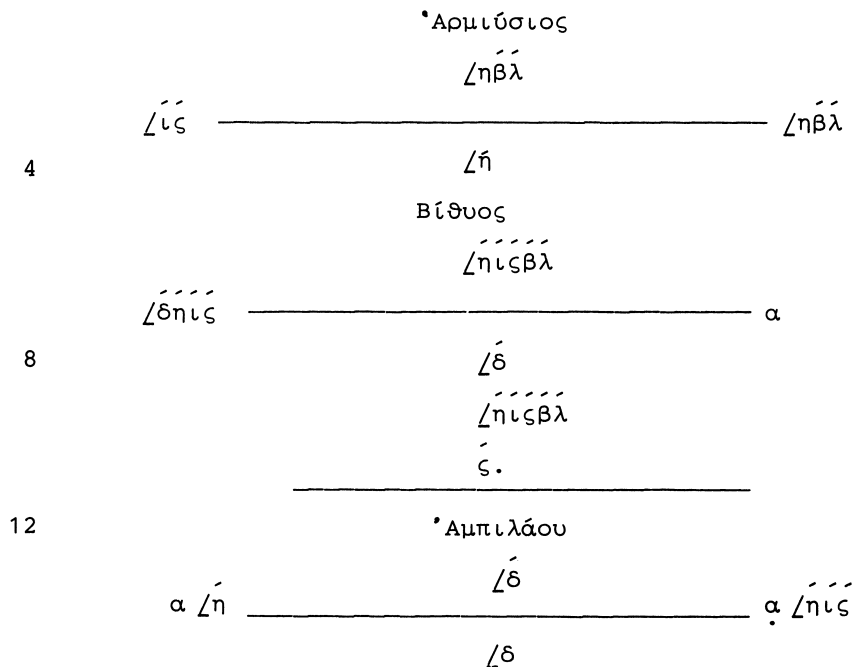
P.Mich.inv.no. 3245. A light-brown papyrus (25.5 x 7 cm.) regularly broken off at all sides. At the top, at the right side, a piece of 3.5 x 1.5 cm. is missing. At approx. 2 cm. from the right border there is a $\kappa\omicron\lambda\lambda\eta\sigma\iota\varsigma$. The text which possibly originates from the Arsinoite nome (cf. note to line 31) has to be dated to the IInd century B.C. We are probably dealing with a draft. This becomes clear from the corrections, from the fact that the calculations of the pieces of land are written on both sides, and from the fact that on the verso another text (upsidedown in relation to the first text) has been added.

Lines 1-25 are dealing with calculations of quadrilaterals according to the usual method: $(a + c) : 2 \times (b + d) : 2$ (cf., e.g., N.Lewis, *Greeks in Ptolemaic Egypt*, Oxford 1986, 112f.). Cf. the Excurs following the publication of the text.

Lines 26-38 (written upsidedown in relation to lines 1-25) contain an account which due to fading away of some parts of the text and to some doubtful readings has to remain unclear. These lines may have been written by a different hand.

RECTO

Taf. Ia



*) We wish to thank Mrs L.C.Youtie who offered P.J.Sijpesteijn this text for publication. We consulted Dr W.Clarysse (Leuven) who gave us, as expected, good advice. W.H.M.Liesker and P.J.Sijpesteijn are responsible for the transcription of the text and the commentary. E.M.Bruins signs for the excurs.

VERSO

Taf. Ib

16]
]ις

[[traces]]
α δ

20 α ι'ς'δ' [[ι]] ————— α ηβλ
α δηις
(γίνεται) α /ηις

24 α η
/ις
α δηις

μζ αν(α) φ

(γίνεται) (τάλαντα) γ 'Εφ

28 εκ τοῦ εχον.α... (τάλαντα) β
(λοιπὸν) (τάλαντον) α 'Εφ
εκεχων εκ τοῦ
Σουχιῆου (τάλαντον) α σκ

32 καὶ παρὰ Κολλούθ(ου)
σπέρματος γα.....
(γίνονται) ψπ ἐργατῶν ..
(γίνονται) π

36 καὶ παρὰ Πετεάσιτος
(δραχμᾶς) 'Α, (γίνεται) εἰς τὸ αὐ(τὸ) <(τάλαντον)> α 'Β π
(λοιπαὶ) 'Αφκ.

Notes:

2) At the left border traces of one (possibly two) deleted letter(s).

/ηβλ: little vertical strokes above most numeral-letters characterise them as fractions (above η this stroke has been left out, probably by mistake. Also in line 3). This is normal practice (Cf. V.Gardthausen, *Griechische Palaeography* II, Leipzig 1913, 373). The scribe consistently writes βλ instead of λβ (cf. PLBat. 21, Chapter VII for other examples).

3) in front of /ις there is a small probably meaningless oblique.

2-4) The surface of Harmiysis' plot is not given. Its surface amounts to 4'8'128'256'512'1024'2048'4096' aroura. This, by *skm* (vide infra) 4096' equal to 4'8'64', has been rounded up to 4'8'16' aroura (cf. lines 9 and 22).

6-9) The exact surface of Bithys' piece of land is 2'8'16'64'128'2048' aroura which has been rounded up to 2'8'16'32' aroura.

10-11) It looks as if the scribe started with the measurement of another plot of land but did not finish it, since he made a mistake (he also left the name of the owner out). The letter after the ζ may be an omikron with a

horizontal above it. This may stand for $\delta\mu(\omicron\omega\varsigma)$ written at the wrong place (cf., however, lines 13 and 15).

12) $\text{'}\alpha\mu\pi\iota\lambda\acute{\alpha}\omicron\upsilon\text{'}$: read $\text{'}\alpha\mu\phi\iota\lambda\acute{\alpha}\omicron\upsilon\text{'}$. The proper name $\text{'}\alpha\mu\phi\acute{\iota}\lambda\alpha\omicron\varsigma$ is not yet listed in the papyrological onomastica.

13-15) Again the surface of Amphilaos' plot of land is not given. The surface amounts to $1\ 8'16'32'64'128'$ aroura which again may have been rounded up to $1\ 4'$ aroura.

13) $\alpha\ \angle\eta\iota\varsigma$: α and δ resemble each other very much in the present hand, but an alpha (probably corrected from \angle) is more likely (also in view of the lengths of the other sides).

16-17) These lines contain the remains of the length of one side of a parcel and probably of its total surface. If this assumption is correct it proves that the papyrus is incomplete at the top.

18) This line is washed off. It probably contained the (wrong ?) name of the owner of the plot of land of which the lengths of the sides are given in lines 19-21.

19-22) The number of the length of the west side of the plot of land involved has been corrected. It looks as if the scribe wrote at first $\alpha\ \iota\varsigma$. Then he seems to have corrected the ς to a δ and may-be he added an iota planning to continue with ς and to delete the original ι . However, he seems to have changed his mind again. He deleted the iota after δ and wrote ς above the δ (corrected from the original ς). This resulted in the strange $\alpha\ \iota\ \overset{\varsigma}{\delta}[[\iota]]$. ($1\ 4'16' + 1\ 8'32'$) : $2 \times (1\ 4' + 1\ 4'8'16')$: $2 = 1\ 2'8'32'512'2048'$ aroura. According to the usual practice in this text this has been rounded up to $1\ 2'8'16'$. If the length of the west side were only $1\ 16'$ (i.e. $\alpha\ \iota[[\delta]]\ \overset{\varsigma}{\delta}$) the surface of the plot of land under consideration would only be $1\ 4'8'16'32'64'256'512'2048'$ which, even rounded up, would hardly result in $1\ 2'8'16'$.

23-25) We do not understand the meaning of these lines. It should be noted that the amounts given in lines 23 + 24 give the amount listed in line 22 and the amount given in line 25 is the same as the length of the south side of the plot of land calculated in lines 19-21.

26-27) D.Hagedorn drew our attention to the fact that 47×500 (drachmae) = 23,500 (drachmae) = 3 talents, 5,500 (drachmae). He, therefore, proposes to read in line 26 $\mu\zeta\ \acute{\alpha}\nu\alpha\ \phi$. Although the form of the ϕ in line 27, 29 and 38 differs from the one in line 26 and looks more like a ψ (cf., however, the shape of that letter in line 34) Hagedorn's observation is absolutely correct.

28) $\acute{\epsilon}\kappa\ \tau\omicron\upsilon\ \epsilon\chi\omicron\nu.\alpha\dots$: instead of $\acute{\epsilon}\kappa$ a reading $\epsilon\lambda\varsigma$ (here and in line 30) is not to be excluded (cf. $\epsilon\lambda\varsigma$ in line 37. The κ in lines 32, 36 and 38 is differently formed). A connection with the verb $\acute{\epsilon}\chi\omega$ (the compound $*\acute{\epsilon}\kappa\acute{\epsilon}\chi\omega$ is not attested) seems excluded, since nothing is received but on

the contrary spent (3 tal. 5,500 dr. [line 27] less 2 tal. [line 28] gives 1 tal. 5,500 dr. [line 29]). Non liquet!

31) Σουχιήου: read Σουχιείου. A Σουχιεῖον is a sanctuary of the crocodile god Suchos. The mentioning of a Σουχιεῖον here may point to the Arsinoite nome as place of origin of the present papyrus (cf. G.Ronchi, *Lexicon theonymon rerumque sacrarum et divinarum ad Aegyptum pertinentium* V, Milano 1977, 1012-1013 [and ff.]).

33) The kind of seed may have been mentioned at the end of this line.

34) At the end of this line the number of labourers was perhaps given.

36) Πετεάσιτος: the proper name Πετεᾶσις is not listed in the PP VII (Index Nominum). In P.Achmim 9, 129 U.Wilcken read Πετεμ() instead of Πετεασ() (cf. BL II.2.10). In Archiv 10, 1932, 266 Wilcken writes: "Da wir einen Gott Asis nicht kennen, bezweifle ich Πετεᾶσ(ιτος) auch in Z.81 (of P.Achmim 9)." The proper name Πετεᾶσις occurs, however, in P.Lond. II 257, 106 (p.22) and 272 (p.28) of A.D. 94.

37) εἰς τὸ αὐ(τὸ) <(τάλαντον)>: D.Hagedorn, referring to the shape of the sign for (τάλαντον) in lines 27, 29 and 31, suggested to us that (τάλαντον) has to be read instead of αὐ(τὸ). He would then transcribe εἰς τὸ (αὐτὸ) (τάλαντον). But we are not acquainted from Ptolemaic texts with a phenomenon comparable with ἐπ() = ἐπ(ὶ τὸ αὐτό) known from many texts of the Roman period. We, therefore, dare not propose this solution.

38) In line 31 there is a question of 1 tal. 220 dr. This amount + 780 dr. (line 34) + 980 dr. (line 35) + 1,000 dr. (line 37) make 1 tal. 2,980 dr., i.e. the total given at the end of line 37. If one subtracts 1 tal. 2,980 dr. from 1 tal 5,500 dr., i.e. the remainder given in line 29, the result is 2,520 dr. Perhaps we should correct the remainder given in line 38 to Βφκ.

EXCURS on the method of calculating areas of quadrilaterals.
Before discussing the numerical part of the Michigan papyrus published above two remarks have to be made:

1) It is still a widely spread opinion that Egyptian arithmetics - especially in land surveying - is complicated and of low level. In fact the method used is the same as that in the modern binary computer. All parts are obtained by iterated halving and adding. If this leads to a long series of parts one can greatly reduce the length by the *'skm*,¹⁾ the "completion". Suppose we have the quantity 13 2'4'8'32'64' then adding 16'64' makes the whole series disappear, beginning from the right hand side 64' + 64' = 32', 32' + 32' = 16' - - -, and the result is 14. Thus the quantity is equal to 14 minus 16'64'.

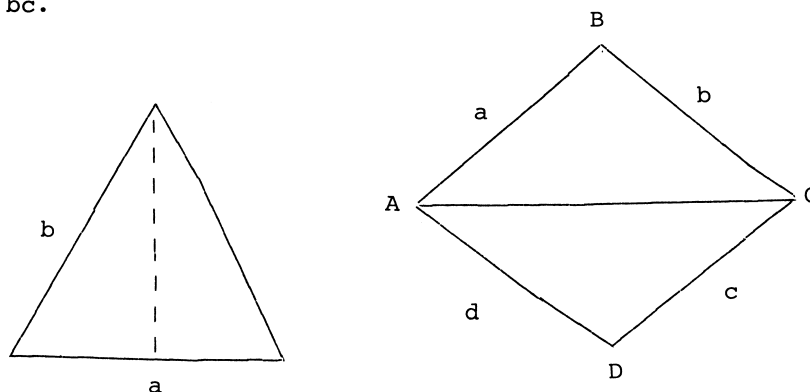
2) Fields are - as in all ancient and medieval societies - measured with the "agrimensor formula" which - except for the rectangle - gives too great

1) Egyptian *km* means "to be complete", the causative *'skm* "cause to be complete".

a value. A simple derivation is the following:

a) the area of a triangle is less than or equal to the half of the product of the lengths of two sides. Equality holds true only if the two sides are at right angles,

b) a quadrilateral ABCD having the lengths of the sides (cyclically) a, b, c, d can be divided in two ways by a diagonal. The diagonal AC leads to: $\text{area} \leq 1/2 ab + 1/2 cd$. Equality holds true only if the angle B and the angle D are right which means that the quadrilateral is inscribed in a circle with diameter AC. In the same way the other diagonal BD leads to the area $\leq 1/2 ad + 1/2 bc$.



A fair approximation is obtained by taking the average of the two results: $\text{area} = 1/4 (ab + ad + cd + cb) = 1/4(a(b + d) + c(b + d)) = 1/2(a + c) \cdot 1/2(b + d)$. Equality holds true only if all angles are right, i.e. the quadrilateral is a rectangle.

Since the measurement of the fields was mostly connected with taxation, the too great value for the area measured did not alarm the authorities.

Turning now to the Michigan papyrus published above we read that the pairs of opposite sides of Harmiysis' field (lines 2-4) are indicated as 2'8'32'; 2'8' and 2'16'; 2'8'32'. In order to have one half of the sum one has to halve all quantities occurring only once and to retain a quantity occurring twice. The area in this case is obtained from the product of 2'8'64' x 2'16'32'64'. Now let us follow first the direct multiplication, leading to the sum of twelve partial products:

4'	32'64'128'
16'	128'256'512'
	128' 1024'2048'4096'
+ 4'8'	128'256'512'1024'2048'4096'

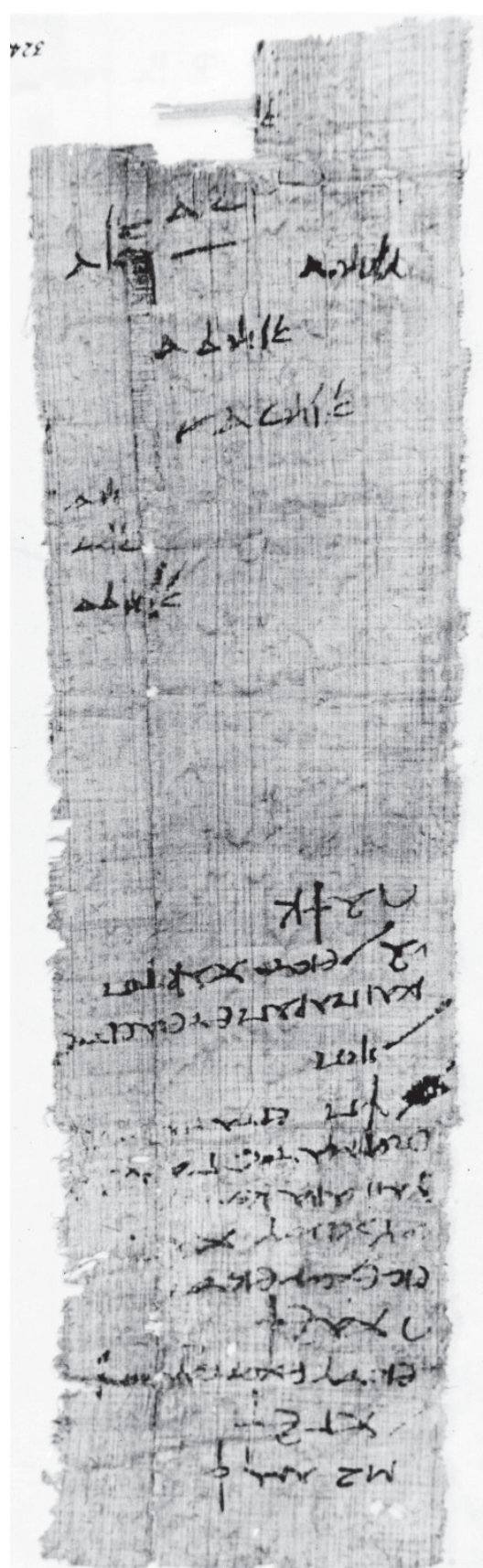
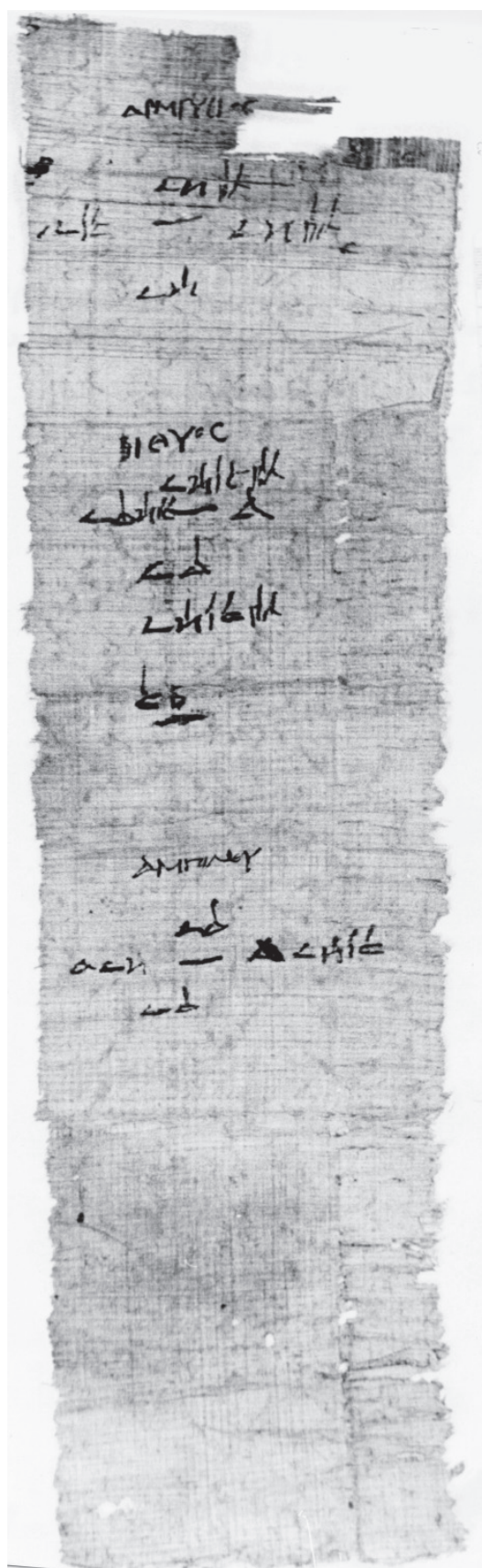
The full result leads to $\overset{\text{'}}{skm}$ 4096' and the result: 4'8'64' (- 4096'). If, however, we apply the $\overset{\text{'}}{skm}$ immediately this leads to: 2'8'64' x (2'8' - 64') = (2'8')² - (64')². Since 64' squared can be neglected, the agrimensor can calculate the surface by heart: 4'8'64'. This thus simple arithmetics

is perhaps the reason that the area of Harmiysis' field is not indicated: one sees it immediately from the data applying the *skm*. The total surface of Bithys' field (lines 6 - 9) is given. We have for the pairs of opposite sides 2'8'16'32'; 2'4' and 2'4'8'16'; 1. This leads to the product: 2'8'16'32'64' x 2'4'8'16'32'. Instead of computing a long series of twenty-five partial products the *skm* leads to: $(2'4' - 64') \times (1 - 32') = 2'4' - 64' - 64' - 128' + 2048'$. Neglecting the last two terms we have the result: $2'4' - 32' = 2'8'16'32'$, i.e. the very result for the surface of Bithys' field as obtained by the scribe.

As regards Amphilaos' field (lines 13 - 15): the pairs of opposite sides are: 2'4'; 2'4 and 1 2'8; 1 2'8'16'. The area is obtained by $2'4' \times 1 2'8'32' = 2'4'16'64' + 4'8'32'128'$ which by *skm* 128' is $1 4' - 128'$, written in full: $1 8'16'32'64'128'$. The reason that the area of Amphilaos' field is not indicated may be the same as the one given above for the field of Harmiysis.

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Landberechnung und Abrechnungen (P.Mich.Inv. 3245 Recto und Verso)