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A CATALOGUE OF TABLES

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The following catalogue is the completion of a preliminary list in *ZPE* 53 (1983) 263-4; it has been abstracted and abbreviated from my book *The Mathematics of Plato's Academy, A New Reconstruction* (Oxford: Clarendon Press, 1987), to which I refer for the discussion on pp. 221-279. It contains all the published examples I have been able to locate of arithmetical tables in Egyptian (hieratic and demotic), Coptic, and Greek. They are identified as being written on papyrus (P.), ostraca (O.), writing tablets (WT.), waxen wooden tablets (WWT.), or as graffiti (G.). The texts are grouped into division tables, multiplication and addition tables, and tables of squares, and are then arranged alphabetically within their language group; museum inventory numbers are distinguished from catalogue numbers; the usual name of a text is set in bold type; if a text appears in Pack^2, its reference number there is given; where a date is assigned or known, it is given; a note about the publication may be given; and each entry contains the briefest description of the contents. The catalogue is restricted to systematic tables; there are many other published examples of exercises involving calculations with, or verifications of, arithmetical expressions which have not been included here.

The main conclusion about ancient numerical practice, argued in my book, is that we have no good evidence for the use of our conception of common fractions, $^m/_n$, in Egyptian or early Greek mathematics. Therefore, it is proposed there, any modern nomenclature or notation that presupposes such a conception should be avoided in most discussions of their mathematics. For example, the sequence of 'parts' is here written as 3", 2', 3', 4', ..., and I recommend that, apart from the first term, it be read 'the half, the third, the quarter, ...'; this in place of the usual name of 'unit fractions', the usual translation as $^2/_3$, $^1/_2$, $^1/_3$, $^1/_4$ ..., and the usual description as 'one half, one third, one quarter, ...'. But I see no objection to the use of such modern notations in translations of texts where this specialised issue, of understanding the details and implications of the numerical practice in the original, is not in question.

I now propose that the very common tables expressing division as sums of parts (examples 1-45, below) should be called 'division tables', rather than the range of names (tables of fractions, tables of multiples, etc.) by which they are generally known. This will, I hope, be more acceptable than the clumsy name, 'tables of parts', that I proposed in *ZPE* 53.

More tables containing interesting variations, will appear in the forthcoming R. Pintaudi & P.J. Sijpesteijn, *Tavolette lignee e cerate della Biblioteca Apostolica Vaticana*, with appendices *Quatre cashiers scholaires* by P. Cauderlier and *A tablet from the Pierpont Library* by R.S. Bagnall. I would be grateful to be informed of any further or omitted material.

I would like to thank the many scholars, especially R.S. Bagnall, B. Boyaval, W. Brashear, A. Bülow-Jacobsen, P. Cauderlier, W.E.H. Cockle, B.R. Goldstein, L. Koenen, P. Mertens, P.J. Parsons, T.S. Pattie, R. Pintaudi, P.J. Sijpesteijn, and Sir Eric Turner, who have provided information and references.

Division Tables

Egyptian, Hieratic

1. **P. Kahun 8.** A fragmentary collection of mathematical problems and a '$^2/_n$-table', with verifications, for n = 3, 5, 7, ... 19, & 21, identical with the expressions in the Rhind Mathematical Papyrus; photograph.

2. **Rhind Mathematical Papyrus,** B.M. [inv.] 10057 and 10058, and a fragment, no. 37. 1784E, in the Brooklyn Museum; copied c. 1575 BC by Ahmose from an archetype some 300 years older. An extensive collection of 86 mathematical problems, a table of 10', and verifications of nth parts of 2 for n = 3, 5, 7, ..., 99, & 101. This text, especially the '$^2/_n$-table', has provoked an enormous literature (bibliography to 1929 in A.B. Chace, *The Rhind Mathematical Papyrus* (1927-9, abr. repr. 1979) abridged and extended randomly in the 1979 reprint), but with only occasional references to other division tables. Photographs and facsimiles in Chace, *op. cit.*, G.R. Robins & C. Shute, *The Rhind Mathematical Papyrus* (1987), and elsewhere.

3. **O. Sen-Mūt 153.** Fragments of a table of 7' with auxiliary red numbers; photograph.

Egyptian, Demotic

4. (P.) B.M. [inv.] 10794, Problems 66 and 67 in **R.A. Parker,** ***Demotic Mathematical Papyri*** (1972) 72-3. Tables of 90' & 150', with entries from 1 to 10; photograph.

5. O. Private collection, in **G. Belli & B. Costa, "Una tabellina arithmetica per uso elementare scritta in demotico",** *Egitto e Vicino Oriente* 4 (1981) 195-200. Table of 2'; photograph.

6. P. Unidentified example, in **E. Revillout,** ***Mélanges sur la métrologie, l'économie politique, et l'historie de l'ancienne Egypte*** (1985), pp. LXIX-LXXIII. (Revillout describes it as "un papyrus mathématique qui m'a été communiqué par l'Exploration Fund".) Tables of 7', 8', ..., 14', & 15'.

Coptic

7. (P.) B.M. MS 528, in **W.E. Crum** ***Catalogue of Coptic Manuscripts in the British Museum*** (1905), and J. Drescher, "A Coptic Calculation Manual", *BASC* 13 (1948-9) 137-60; c. AD 900. A palimpsest parchment codex; the later Coptic text contains multiplication tables (see no. 47, below), mathematical exercises, and division tables for 2', 3", 3', 7' [*sic*], 4', 5', 6', 8', 10', 12', 15', 16', 20', 24', & 48'; only the first few entries of the tables for 2', 15', & 48' are transcribed. Described as "identical with those in the Greek mathematical papyrus of Achmîm", no. 12, below.

8. P. Cambridge University Library [inv.] T.-S. Ar 39:380, in **B.R. Goldstein & D. Pingree, "More Horoscopes from the Cairo Geniza",** *PAPS* 12 (1981), on 186-9. Two folia containing multiplication tables (see no. 48, below) and tables for 12', 24', & 48'; photograph.

9. **O. Crum, 480.** A fragmentary potsherd table of 31'. Crum was unable to identify its purpose in 1902 (but see no. 7, above). There is a description in K.H. Sethe, *Von Zahlen und Zahlworten* (1916), 71-2, but some of his restorations are clearly wrong; see W. Brashear, "Quisquiliae", *BASC* 26 (1984) 19-22.

10. (P.) Strasbourg Bibliothèque Universitaire [inv.] 4110, same publication as no. 8, above, on 176-7. Table of 24', a marginal annotation to an astrological text; photograph.

11. **O. Wadi Sarga 24-28** (= *Coptica* III 24-28); also see W. Brashear, "Quisquiliae", *BSAC* 26 (1984) 19-22. Five fragmentary potsherds with brief traces of division tables for 7' (3 examples), 11', 25', 49', & one unidentified table.

Greek

12. **Achmîm Mathematical Papyrus** (= P. Cair. [inv.] 10758), Pack^2 2306, in J. Baillet, *Mémoires de la Mission Archéologique Francaise* ix 1, (1982); VII AD. Leather-covered papyrus codex containing division tables for 3', 3", 4', ... 19', & 20', and mathematical exercises; complete facsimile.

13. P. Berol. [inv.] 212296, in **W. Brashear, "Greek Papyri: Fractions and Tachygraphy",** *Anagennesis* 3 (183) 167-88; II BC. Fragments of tables of 3' & 4'; photograph.

14. (WT.) Bodleian Gr. Inscription [inv.] 3019 in **P.J. Parsons, "A School-Book from the Sayce Collection",** *ZPE* 6 (1970), on 142-3; late III AD. Seven wooden school tablets containing a wide variety of exercises in Greek and Coptic; three sides contain division tables for 2', 3', 4', 5', 6', 8', 9', 10', & 12'. Only the table of 2' is transcribed.

15. **P. Freib. i 1**; also see W. Brashear, Greek Papyri: "Fractions and Tachygraphy", *Anagennesis* 3 (1983) 167-77; II or I BC. Fragment of a table of 3".

16. (WT.) Louvre [inv.] AF 1196^1, AF 1196^2, AF 1196^3 AF 1197^1, in **B. Boyaval, "Tablettes mathématiques du Musée du Louvre",** *Rev. Arch.* (1973) 243-60 and "Le cahier de Papnouthion et les autres cahiers scolaires grecs", *Rev. Arch.* (1977) 215-30; P. Cauderlier, "Cinq tablettes en bois au musée du Louvre", *Rev. Arch.* (1983) 259-80; and W. Brashear, "Corections à des tablettes arithmétiques du Louvre", *Rev. Ét. Gr.* 97 (1984) 214-17 (all

publications must be consulted; photographs in *Rev. Arch.*); VI AD. Parts of a wooden schoolbook containing multiplication (see no. 54, below) and division tables:

AF 1196^1 side B: table of 9';

AF 1196^2 side B; table of 3", with verifications (or possibly derivations; see nos. 26 & 44, below) of each successive entry, up to 900. Also see AF 1196^3 side B, below;

AF 1196^3 side A: table of 11' and multiplication tables;

AF 1196^3 side B: on the basis of three legible letters, Cauderlier proposes a resoration of the continuation of AF 119^2 side B, a table of 3" with verifications from 1000 up to 10000;

AF 1197^1 side A: table of 14';

AF 1197^1 side B: table of 17'.

17. (WWT.) Louvre [inv.] MND 552, MND 552h, i, k, & l; same publications by B. Boyaval as the previous entry; V or VI AD. Wax tablets containing a collection of school exercises, some arithmetical:

MND 552k: Headed and dated, "Papnouthion son of Iboïs, Mechir 21st" (incorporating a corrected reading by P. Cauderlier; personal communication), a table of 24', with six alternative entries for 2 (of 2 the 24' is 12', or 15' 60, or 20' 30', or 30' 30' 60' [*sic*; this is the only example I know of a repeated part], or 45' 48' 60' 80' 90', and one lost entry).

MND 552i: Again headed and dated Mechir 21st, a fragmentary table of 25', again with duplicated but largely illegible expressions for the 25' of 2.

18. (WT.) Louvre [inv.] MND 551dl, in **B. Boyaval, *CRIPEL* 2** (1974) 270-1 and W. Brashear, "Trifles", *ZPE* 56 (1984) on 64-5; IV-VI AD. Fragmentary table of 3".

19. **P. Mich. III 145**, Pack^2 2309; II AD. Fragments of tables of 23' & 29', and arithmetical problems.

20. **P. Mich. III 146**, Pack^2 2310; IV AD. Long narrow roll (106.75 x 9.2cm) broken at the beginning, incomplete at the end, containing tables of 7' (fragmentary), 8', 9', ... 18', & 19' (heading ἐννεακαιδέκατα to a blank column).

21. **P. Mich. III 147**, Pack^2 2311; early II AD. Fragments of a table of 4'.

22. **P. Mich. XV 686**; II-III AD. Multiplication table (see no. 55, below) and fragmentary division tables including a table of 30' increasing by halves.

23. O. Mich. [inv.] 9733, in **H.C. Youtie, "A Table of Fractions"**, *ZPE* 18 (1975) 17-19; first half of III AD. Potsherd table of 3".

24. (WT.) **P. Michael. 62**, Pack^2 2308; VI AD? Tables of 2', 3", 3', & probably 4' (now illegible), and, on the other side, mathematical problems. More complete publication in D.S. Crawford, "A mathematical tablet", *Aegyptus* 33 (1953) 222-40.

25. (WT.) **MPER N.S. XV 154**; VII AD. Multiplication tables (no. 59, below) and a table of 2'; photograph.

26. **MPER N.S. XV 156**, first published in P.J. Sijpesteijn, "Wiener Mélange", *ZPE* 40 (1980) on 97-8, and re-edited in W. Brashear, *Enchorica* 12 (1984) 1-6; V-VI AD. Fragment containing only multiplications, but very possibly coming from a table of 9' with verifications (or calculations) similar to those in Louvre AF 1196[2] side B and P. Würzburg K 1024 (nos. 16 & 44, here); photograph.

27. **MPER N.S. XV 158**, III AD? Minute fragment of a table of 3"; photograph.

28. **MPER N.S. XV 159**, first published in P.J. Sijpesteijn, "Wiener Mélange", *ZPE* 40 (1980) on 98-9; III or IV AD. Table of 3", followed by a financial calculation; photograph.

29. **MPER N.S. XV 160**; II AD. Tables of 2' & 3'; photograph.

30. **MPER N.S. XV 161**; VII-VIII AD. Tables of 2' & 4', but not written out in tabular form—the only such example known to me; photograph.

31. **MPER N.S. XV 162**; first half of IX AD. Fragment of table of 3"; photograph.

32. **MPER N.S. XV 163**; VI AD. Fragment of tables of 3' & 4'; photograph.

33. **MPER N.S. XV 164**; VI-VII AD. Fragment of tables of 3' & 4'; photograph.

34. **MPER N.S. XV 165**; VI-VII AD. End of table of 5' & table of 3"; photograph.

35. **MPER N.S. XV 166**; II BC. End of table of 4'; photograph.

36. **MPER N.S. XV 167**; II AD. Fragments of tables of 7', 8', & 9'; photograph.

37. **MPER N.S. XV 168** verso; VII AD. Fragment of table of 14'; photograph.

38. **MPER N.S. XV 169** recto and verso; VIII AD, Fragment of tables of 13', 14', 11', 17' & 19'; photograph.

39. (WT.) Meon [inv.] 602, in **P.J. Sijpesteijn, "A Wooden Table in the Moen Collection"**, *Chronique d'Egypte* 56 (1981) 97-101; VI or VII AD. Half a tablet containing part of a table of 7'.

40. **P. Oxy. XXXIII 2656**, first published in E.G. Turner, *New Fragments of the Misoumenos of Menander, BICS* Supplement 17 (1965), on 18-19; IV or V AD. Tables of 13', 14', 15', 16', & perhaps 17' & 18'. These tables are found in a papyrus codex that otherwise carried a lost play of Menander; there would have been space for tables of 10' to 19'. Only the table of 13' is transcribed. The republication as P. Oxy. XXXIII 2656 omits the tables.

41. **P. Oxy. XLIX 3456**; III or IV AD. Tables of 7' & 8', followed by metrological definitions. Only part of each table is transcribed.

42. (WT.) University College [inv.] 36114, Pack[2] 2312, in **H. Thompson, "A Byzantine Table of Fractions"**, *Ancient Egypt* 1 (1914-15) 52-4. Tablet containing tables of 15' & 16'; facsimile. The editor misunderstood the initial entry, but it can clearly be guessed from his drawing and I have checked the original.

43. (WT.)Würzburg [inv.] K 1014, in **W. Brashear, "Holz- und Wachstafeln der Sammlung Kiseleff"**, *Enchoria* 13 (1985) 13-23 and Tafel 1-13; VI-VII AD. Four tablets con-

taining school exercises in multiplication and division. Tables of 6', 3", 5', & 3' (in that order); photographs.

44 (WT.) Würzburg [inv.] K 1024, in **W. Brashear, "Neue Griechische Bruchzahlentabellen",** *Enchoria* 12 (1984) 1-6 and Tafel 1-2; VIII AD. Tables of 3" & 11', with verifications (or calculations) similar to those in Louvre AF 1196^2 side B and M.P.E.R. 156 (nos. 16 & 26, above); photograph.

45 (WT.) **SB III 6219**, first published in G. Plaumann, "Antike Schultafeln aus Ägypten", *Amtliche Berichte aus den Königlichen Kunstsammlungen* (Berlin) 34 no. 11 (1913) 210-23, on 222ff.; VII AD. Tables for 2' & 3", and problems in addition and multiplication; photograph.

Addition and Multiplication Tables

Egyptian, Demotic

46. P. B.M. [inv.] 10520, Problem 54, in **R.A. Parker, Demotic Mathematical Papyri** (1972). List of multiples of 16 from 1 to 16; photograph.

Coptic

47. P. B.M. MS 528 (= no. 7, above). Table, presumably with first page missing, of 7x, 8x, 9x, 10x, 20x, ..., 900x.

48. P. Cambridge University Library [inv.] T.-S. Ar. 39:380 (= no. 8, above). Table of 1x, 2x, ..., 9x, 10x, 20x, ..., 90x, 100x, 200x, ..., 1000x; photograph.

49. G. Monastère de Phoebammon 5 (mural no. 10), 153 (mural no. 186), and possibly 6 (mural no. 12), published by R. Rémondon in **C. Bachatly,** ***Le monastère de Phoebammon dans le Thébaïde***, Tome II: ***Graffiti, inscriptions et ostraca*** (1965), with review by J. Schwartz in *Chronique d'Egypte* 42 no. 83 (1967) 251-4. Tables of 3x & 4x, and an unidentified table; photograph.

50. **O. Wadi Sarga 22 and 23** (see no. 11, above). No. 22: tables of 6x & 7x; no. 23: tables for 7x, a palimpsest, with this the younger text.

Greek

51. **P. Baden IV 644**, complete publication in F. Bilabel, *Berichtigungsliste* II 177-81. Table of 2x, 3x, ..., 9x, 10x, 20x, ..., 90x, 100x, 200x, 300x, 400x, 500x; then 2x, 3x, ..., 9x, 10x, 20x, 30x in another format.

52. (WWT.) Leiden Papyrological Institute, in **E. Boswinkel, "Schulübungen auf 5 Leidener Wachstäfelchen",** *Proceedings of the XIV International Congress of Papyrologists, Oxford 1974* (1975), on 25-28; after AD 212. Five tablets, one containing a table of 1x40 to 10x40.

53. P. Lond. III 737 (p. XXX) published as **MPER N.S. XV 150**; III AD. Table of $n+m$ for $n \leq m$ and $1 \leq n \leq m \leq 9$, then similarly for 10+10, ..., 10+90, 20+20, ..., 20+90; photograph.

54. (WT.) Louvre [inv.] AF 1196[1], 1196[2], and 1196[3] (= no. 16, above);

    AF 1196[1] A: tables of 2x, 3x, ..., 8x;

    AF 1196[2] A: traces of tables, illegible at beginning, for [8x], 9x, 10x, 20x, ..., 50x;

    AF 1196[3] A: tables of 5000x, 6000x, ..., 9000x.

55. **P. Mich. XV 686** (= no. 22 above). Fragments of a multiplication table up to 10000x4.

56. **MPER N.S. XV 151**, the verso of M.P.E.R. I 1; I AD. Addition table in same format as no. 53, above, continuing (with gap for 30+ and 40+) up to 1000+1000; photograph.

57. **MPER N.S. XV 152**; II-III AD. Fragments of tables of multiples and squares. The multiplication table gives 7x8, 7x9, 7x10, 8x1, ..., 8x10, 9x1, ..., 9x10, 10x1, ..., 10x10, followed immediately by the table of squares (see no. 68, below); photograph.

58. **MPER N.S. XV 153**; II AD. Fragment of table of 20x500, 20x5000, 20x6, 20x60, ..., 20x9000; photograph.

59. **(WT.) MPER N.S. XV 154**; VII AD. Tables of 7x1, 2, ...9, then 8x, & 9x, and a table of 2' (no. 25, above); photograph.

60. **MPER N.S. XV 157**; IX-X AD. Fragment of tables of 2x & 3x; photograph.

61. **P.S.I. VIII 958**, Pack[2] 2307; IV AD. Table headed πολυπλάσιμος, starting with some unsystematic entries for smaller numbers, then systematically for 30x, 40x, ..., 90x, 100x, 200x, 300x, ..., 1000x, 2000x, 3000x, & 4000x.

62. (WWT.) **SB III 6215**, first published in G. Plaumann, "Antike Schultafeln aus Ägypten", *Amtliche Berichte aus den Königlichen Kunstsammlungen* (Berlin) 34 no. 11 (1913) 210-23, on 216. Addition table, 8+1 to 8+9; photograph.

63. (WWT.) Würzburg [inv.] K 1013, in **W. Brashear, "Holz- und Wachstafeln der Sammlung Kieseleff"**, *Enchoria* 13 (1985) 13-23 and Tafel 1-13; IV-V AD. Parts of five wax tablets containing school exercises including two simple addition tables; photographs.

64. (WT.) Würzburg [inv.] K 1014; see previous entry and no. 43 for publication details. The multiplications are as follows. Side 1B: 60x1, 60x2, ..., 60x10, then similarly 70x, 80x, 90x, 100x, 200x, and 300x; side 2A: 400x, 500x, 600x, then reverting to 20x, 30x, 40x, 50x; Side 3B (fragment): 700x3, ..., 700x8, 800x8, ..., 900x4, 1000x4, ..., 1000x10, 300x1, ..., 3000x7; photograph.

Tables of Squares

Coptic

65. G. Monastère de Phoebammon 16 (mural no. 97); see no. 49, above. Table of squares for 10, 11, ..., 29, & 30; photograph.

Greek

66. **P.** ***Un livre d'écolier du III^{e} siècle avant J.C.***, ed. O. Guéraud & P. Jouguet (1938) = P. Cairo J.E. [inv.] 65445, lines 216-34. Table of squares of 1, 2, ..., 10, 20, ..., 100, 200, ..., 800; complete facsimile.

67. **P. Haun. III 49**; II BC? Table of squares for 1, 2, ..., 19, 20, 30, & 40; photograph.

68. **MPER N.S. XV 152**; II-III AD. Multiplication table (see no. 57, above) and two tables of squares, first in the form 1 1; 4, 2; 9, 3; ..., 100 10; 400 20; ...; 6400 80; 10000 100; 40000 200; ...; 250000 500; then in the form 1 1 1, 2 2 4; ..., 10000 10000 100000000; photograph.

69. **P.S.I. VII 763**, Pack^{2} 2315; I BC. Fragment of a mathematical lesson, partly in question and answer form. Lines 5 to 11 deal with squares, but not in tabular form: ἄπ]αξ ἓν α, δὶς δύο δ τρὶς τρία ἐννέα θ τετράκις τέσσαρα ιζ πεντάκις πέντε κε up to δεκάκις δέκα ρ (ἄπ]αξ ἓν *supplevi*:]αξεν *edd.*).

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