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The Shape and Size of the Shield in the Inscription from Temple G at Selinus (*IG* 14.268)

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1. Over twenty years ago W.M. Calder III published what has since become the standard edition of the inscription from Temple G at Selinus.[1] Recently scholars have returned their attention to this important text.[2] However, with the exception of the brief calculations by Calder (p. 51), little attention has been paid to the dimensions of the offering. If we accept Calder's hypothesis of a shield offering, some fairly elementary physical considerations allow us to constrain tightly the possible dimensions. In addition, these considerations tend to provide some confirmation of Calder's preferred size. It is hoped that the methodology provided here may aid scholars dealing with similar problems.

2. Calder argues on philological grounds and from analogy that the offering must have been a shield.[3] This does not establish the exact shape. There are essentially three possibilities: (1) an oval ("Boiotian"), (2) a round or circular ("Argive") "*hoplon*" and (3) a Thracian (light) "peltē" shield.[4] The "Boiotian" shield seems to be known only from representations[5] and hence is not

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[1] William M. Calder III, *The Inscription from Temple G at Selinus*, Greek, Roman, and Byzantine Monographs 4 (Durham, NC, 1963), henceforth cited Calder[1], and idem "Further Notes on *IG* xiv 268 and Other Tufa Inscriptions from Selinus," *GRBS* 5 (1964) 113-121, henceforth cited Calder[2]. See the reviews: von der Mühll, *MH* 21 (1964) 244; Oates, *CW* 57 (1964) 378; Pouilloux, *REA* 66 (1964) 213-6; Betts, *CPh* 60 (1965) 288-290; Herrmann, *Gnomon* 37 (1965) 377-380; Roux, *RPh* 39 (1965) 296-7; Woodhead, *CR* 15 (1965) 232-3; Parke, *Hermathena* 102 (1966) 101; teRiele, *Mnemosyne* 19 (1966) 449-451; Woodward, *JHS* 86 (1966) 296-7.

[2] See D. Musti, *RFIC* 113 (1985) 134-157; and R.R. Holloway, *Revue des Archéologues et Historiens d'Art de Louvain* 17 (1984) 7-15. In addition there have been two editions, dependent on Calder[1]: R. Meiggs and D.M. Lewis, *A Selection of Greek Historical Inscriptions* (Oxford 1969) no. 38; and M.T. Manni Piraino, *Iscrizioni greche lapidarie del Museo Palermo* (Palermo 1972) no. 49.

[3] Calder[1], 45-47; Calder[2], 118. Musti's recent preference for a stele is unconvincing: *RFIC* 113 (1985) 145-148.

[4] See M. Greger, *Schildformen und Schildschmuck bei den Griechen* (Diss. Erlangen 1908) 2-16 for Mycenaean and Geometric shield-forms, 16-32 for historical shields; Georg Lippold, *Zu den Schildformen der Alten* (Diss. München 1908), cited Lippold[1], 5-12; and idem, "Griechische Schilde" in *Münchener Archäologische Studien* (München 1909) 399-504, cited Lippold[2] (Lippold[1] = Lippold[2] 399-429), esp. 403-410; Snodgrass, *Early Greek Armour and Weapons from the End of the Bronze Age to 600 b.c.* (Edinburgh 1964), henceforth cited Snodgrass[1], 36-38; idem *Arms and Armour of the Greeks* in *Aspects of Greek and Roman Life*, ed. H.H. Scullard (Ithaca, NY 1967), henceforth cited Snodgrass[2], 53-55, 58, 67-9, 75, 95, 104-5. Three other shield-forms may be discounted on chronological grounds: the Mycenaean figure-eight, the Mycenaean tower (half-cylinder), and the Geometric "cut" oval (similar to the figure-eight). In fact round shields existed in the Mycenaean and Geometric periods (see Greger, Lippold and Snodgrass), but that does not affect the discussion (unless to *increase*

likely in an actual offering. The Thracian "peltē" fails as it was foreign to the Greeks till ca. 420 BC,[6] and one would scarcely make a light shield the model for a gold offering of at least 112 kg (see below on the weight). The "*hoplon*" was the most popular shield in the classical period,[7] and appropriate for Selinus founded by Argos' neighbor, Megara. Thus the most likely shape of the offering was round, as already assumed by Calder ([1]51).

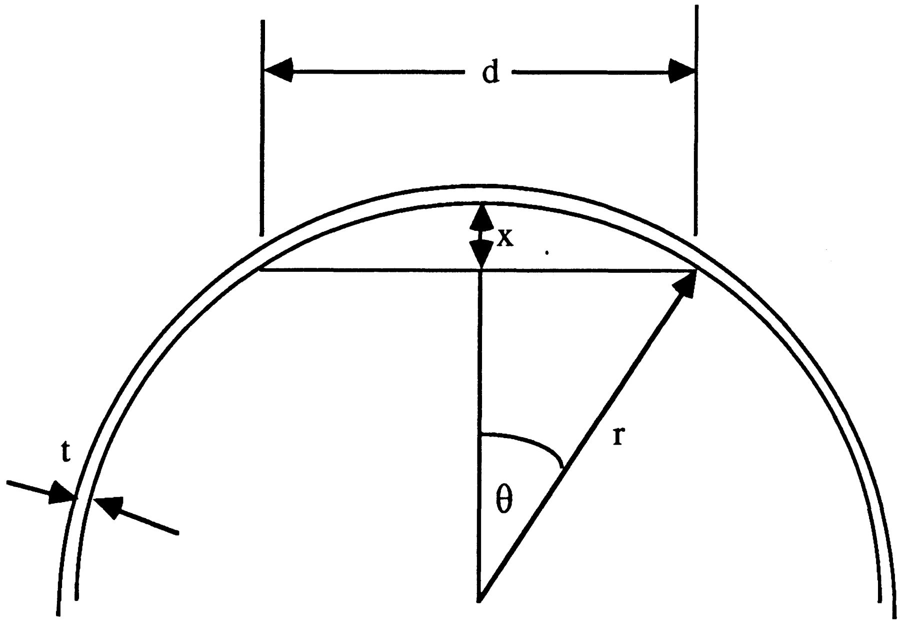

Figure 1: geometry of "spherical cap" model of *hoplon* shield

Calder assumes a disc shape, but shields were rarely flat: a bellied shape is more able to resist assault. A better but still fairly straightforward mathematical approximation to the shape is a

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the likelihood of the round shape in our offering). These round shields differed only in the strap and grip arrangements: see Snodgrass[2] 61 (§ F).

[5] See Snodgrass[2], 55 and Snodgrass[1], 58, following T.B.L. Webster *From Mycenae to Homer* (London 1958) 169-170.

[6] See Snodgrass[2], 110, on the introduction of the peltē; in general see Fr. Lammert *RE* 19 (1938) 406.

[7] See Snodgrass[2] 49, 60, 95 and 105 and Lippold[2] 488, and 504 on the widespread persistence of this hoplon shield form. Shields have been found in various sites, some of which are inscribed dedications. For example, Shear, *Archaiologikē Ephemeris* 100.1 (1937) 140-143 (the inscription reads: 'Αθηναῖοι | ἀπὸ Λακεδ|αιμ[ον]ίων | ἐκ [Πύ]λο(υ), cited by Snodgrass[2], 105, pl. 19: a *hoplon*; and Furtwängler, *Die Bronzefunde aus Olympia und deren kunstgeschichtliche Bedeutung*, Abh. Berlin (1879) no. 4, 80 (the inscription reads Ταργεῖοι ἀ[νέθεν ...] : a *hoplon*. W.H.D. Rouse, *Greek Votive Offerings* (Cambridge 1902) records numerous dedicated shields known from inscriptions, Pausanias or the Greek Anthology. Some are referred to only as ἀcπίc (and for the ἀcπίc the model was a round shield according to Lippold[2] 459): 107, n. 7 (ἐπίχρυcoc); 114, n. 6 (probably a *hoplon* as a panoply was dedicated) or 226, n. 2 (probably a *hoplon* as a hoplite is depicted). A few are peltai: 110, n. 14 and 395. Most are hopla or at least round: 124, n. 11; 262, n. 26; 407; 114, n. 10 (a φιάλη, hence round), or 408 (a cάκοc, described as round in Aisch. *SeTheb* 540, 642, hence of the possible types no doubt a round shield) e.g.g.

"spherical cap" (see Figg. 1 and, below, 2). The volume of such a cap is the difference in volume of the outer and inner spherical sectors, or equivalently the outer and inner spherical zones (represented in either case by radii of curvature r + t and r): see Appendix. We obtain:

$$V = \frac{\pi t\,(4x^2 + d^2)}{4} \qquad (1)$$

where we have expressed the result in terms of d and x as they are directly measurable (while r and θ are not), and note that this is valid only for t small compared to d (not a great restriction). Note that for a flat shield (x = 0) this correctly gives the volume of a disc. Archaeological authorities do not give x-values for the *hoplon*, but I would estimate $^d/_x$ from the available photographs as ca. 8 ± 2, while typical d-values are 0.80 to 1.00 m.[8] The t-value of the typical *hoplon* does not matter for our purpose (as we will see) so long as our model shield has t small compared to d.

3. What were the dimensions of the shield? If 60 T (Talents) gold is right we have about 1572 kg gold (see Calder [1], 51), if Schübring is right (value of gold object expressed in Talents of silver), about 112 kg.[9] These numbers and the density of gold (18.85 gm/cm^3),[10] give us the volume of gold used. We have about 83,400 cm^3 (or if Schübring is right, about 5960 cm^3). If in formula (1) above we set x = d/8 (see § 2 above), we need only the d (diameter) and the t (thickness) of the shield offering. Calder assumes, e.g., a diameter of 8 feet (i.e., 243.8 cm), which would give, for a disc ($V = \frac{\pi t d^2}{4}$), a thickness of 1.79 cm (for the 60 T shield). With the same diameter, the spherical cap shield (x = $^d/_8$) would have a thickness of 1.68 cm.

But there is a natural way to determine a small set of probable diameters and thicknesses. The numbers found above for t are close to one (Ionic) daktyl (1 *d* = ca. 1.85 cm).[11] It is usual to

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[8] Inferable from Greger (above, n. 4) 18, stated in Snodgrass[2], 53 and more precisely in Snodgrass[1], 64 and Lippold[2] 443. See also F. Lammert, *RE* 2A (1923) 424. The photographs are from Shear (above, n. 7) and Snodgrass[2] pl. 19. The d/x value of 8 ± 2 and the diameter of 0.8 to 1.0 m give a bulge of 11 ± 3 cm, roughly the thickness of an arm (not inappropriate). In Fig. 1, $^d/_x$ is ca. 6.4.

[9] I use 436.7 gm for the Attic/Euboic mina or 26.20 kg per Talent, see C.F. Lehmann-Haupt, *RE* Suppl. 8 (1956) 794 and W.Becher, *RE* 15 (1931) 2244-2245. The value varies in modern authorities from 426 and 2/3 to 436 and 2/3 gm per mina (*RE* Suppl. 8, 801) or more (*RE* Suppl. 8, 801-808, and C.F. Lehmann-Haupt *RE* Suppl. 3 [1918] Table on 611-614, entry 11). For the 14:1 ratio of gold to silver, I accept Calder[1] 51, following A.W. Gomme *Thucydides* II (Cambridge 1956) 25, *ad* 2.13.5, though this ratio did vary: see H. Stein, *Herodotos* II[4] (Berlin 1893) *ad* 3.95 (pp. 110-111).

[10] See C. Hammond in *Chemical Rubber Company Handbook of Chemistry and Physics*[63] (Boca Raton, FL, 1982) p. B18, specific gravity = 18.88 at 20° C. The density of water at 20° C = 0.99823 gm/cc, thus the *density* of gold is (0.99823)(18.88) or 18.85 gm/cc (at 20° C). In the 66th edition (1985) Hammond returns to the 19.31 gm/cc value found elsewhere (cp. p. B98 and J.A. Bard in *Metals Handbook*[9] II, ed. W.H. Cubberly *et al.* [Metals Park, OH, 1980], p. 680). This (ca. 2.4 %) variation is comparable to the variation in gm/mina values (above, n. 9). I use the lower density as ancient gold (being impure and cast) could not attain the modern theoretical maximum density. Using the larger density would decrease the volume (by ca. 2.4 %, and hence the maximum diameter (by ca. 1.2 %), and so the d/x value in Table 2 would become larger (i.e., a flatter shield).

[11] Again the absolute value varied. See Fr. O. Hultsch, s.v. "Daktylos (3)" *RE* 4 (1901) 2020-2021. The value 1.85 cm is roughly the average of the shorter (Ionic) daktyl; the longer (Doric)

manufacture objects to whole measures rather than to fractions thereof: i.e., we would expect a thickness of one or two daktyls, not 0.97 or 0.91 *d* as above. Thus we may estimate the t and d by assuming round figures in both cases, perhaps t = 1 daktyl and d = 8 (Greek) feet (*p*), as Calder very nearly does.

We can use the known dimensions of the various temples at Selinus[12] to determine the size of foot (πούς) in use, and hence determine the daktyl (16 *d* = 1 *p*). We convert the dimensions in modern feet to Greek feet using the known approximations (Doric *p* = ca. 1.072 feet, Ionic *p* = ca. 0.967 feet.[13] The we adjust the values in ancient feet (by no more than, say, one percent) to make them as "round" as possible. Finally we choose in each case the ancient foot-measure (Ionic or Doric) which is the rounder figure[14] (multiples of 100 chosen over those of 50, of 50 over 25, of 25 over 10, of 10 over 5, of 5 over 2). Table 1 presents the results, and shows that for these temples the Ionic foot was probably used. The average ratio of modern feet to Ionic feet in the Table omitting the apparently anomalous Temple A) is 0.968 ± 0.008, which gives 29.5 ± 0.2 cm per Ionic foot or 1.84 ± 0.02 cm per Ionic daktyl, in these temples at Selinus.[15]

Now we set the thickness of the offering to a few reasonable values (in both the 60 T case and the case suggested by Schübring), and adjust the $^d/_x$ ratio (flatness of shield) to obtain "natural" values of the diameter. See Table 2. Note that d must be close to d(max) in order that $^d/_x$ not be too small ("too bellied"). As t gets smaller d increases until the shield is too thin to hold its own weight ($^t/_d$ too small), while as t gets larger d decreases until the shield is so thick and heavy that it is unaesthetic and hard to make ($^t/_d$ too large). The width of the adyton would seem to be ca. 20 (Ionic) *p*,[16] so that diameters larger than ca. 15 (Ionic) *p* would crowd the space, while diameters smaller than ca. 4 (Ionic) *p* would leave too much blank space (and give a shield *smaller* than the

would be ca. 2.05 cm. Cp. A.N. Sherwin-White s.v. "Measures" *OCD*² (1969). As we will see, the daktyl appropriate here is the shorter.

[12] They are given in H. Berve and G. Gruben, *Greek Temples, Theatres and Shrines* (New York 1962) 421-432, cp. W.B. Dinsmoor, *The Architecture of Ancient Greece* (London/New York³ 1950) 78-84, 99-100. Measurements for the small Temple B I do not find, while Gruben describes Temple F as "extremist ... peculiar ... freakish" (426). Thus it is excluded (and results from it are ambiguous at best). I do not use either the column diameter or spacing ("interaxial") of the temples as they vary greatly in a given temple (Berve and Gruben, 424 and 430).

[13] Derived from the daktyl values in n. 11 above. There does not seem to be an article "Πούς" or "Fuß" in the *RE*, though Wilhelm Becher s.v. "Pes" *RE* 19 (1937) 1085.58-1086.10 evidently expected an article "Πούς" (B. Kötting *RAC* 8 (1972) 736.40 s.v. "Pes" cites only Becher's "Pes", and *Der Kleine Pauly* offers only "Pes", see Henri Chantraine in v. 4 (1972) 665.51-6.2). See also F. Hultsch, *Griechische und Römische Metrologie* (Berlin² 1882) 30-1, 44-5, 697.

[14] The preference for rounder dimensions would apply *omnibus paribus* to the offering as well as to the temples. For a contemporary example of this preference, cp. Herodotus as noted by P. Keyser *CJ* 81 (1986) 231, n. 4.

[15] Note that this also gives steps of "round" size. Calder² 117, gives the height of the first step as 45 cm (i.e., 1.5 Ionic feet), the height of the second as 24 cm (i.e., 13 Ionic daktyls), and the width of the second as 74 cm (i.e., 2.5 Ionic feet). The height of the inscription is given as 1.77 m, or exactly 6 Ionic feet.

[16] From the scale drawings at Dinsmoor 79 and Berve-Gruben 429. That in Berve-Gruben is larger and easier to use.

Table 1
Dimensions of Temples at Selinus

Demeter	propylaeum		28' 10. 5"	28' 6. 5"	
		Doric	27	27	
		Ionic	*30.*	*30.*	
	megaron		31' 2.75"	66' 11.5"	
		Doric	29	62	
		Ionic	*32*	*70.*	
Temple A	stylobate		52' 11"	132' 2.5"	
		Doric	*50.*	123	
		Ionic	55	127	
Temple C	stylobate		78' 6. 5"	209' 0.75"	
		Doric	73	195	
		Ionic	*80.*	215 (cp. Temple D stylobate)	
	cella		34' 1.5"	136' 3.75"	
		Doric	32	127	
		Ionic	35	*140.* (note: $^{140}/_{35} = {}^{4}/_{1}$)	
Temple D	stylobate		77' 6.25"	182' 8.125"	
		Doric	72	170.	
		Ionic	*80.*	190. (cp. Temple C stylobate)	
Temple E	stylobate		83' 0.825"	222' 2.825"	column height 33' 3.625"
		Doric	78	208	31
		Ionic	*85*	*230.*	*34*
	cella		46' 4.625"	162' 0.125"	
		Doric	43	150.	
		Ionic	*48*	168 (note: $^{168}/_{48} = {}^{3.5}/_{1}$)	
Temple G	stylobate		164' 3.25"	361' 2. 5"	column height 48' 2.75"
		Doric	153	335	45
		Ionic	*170.*	*375*	*50.*

In Temple E, cella length, the choice between the Doric 150 (ordinarily the "rounder" figure) and the Ionic 168 is complicated by the preference for the Ionic 48 in the width, and by the exact ratio of $^{168}/_{48} = 3.5$; underlined numerals indicate the value chosen as rounder.

Summary: Ionic foot preferred in 15 cases, Doric in one, uncertain in four.

Table 2
Possible "Round-figure" Thickness and Diameters

t (I. dak.)	max. d (I. ft.)	d (I. ft.)	$^d/_x$	remarks
(60 T shield, 83,400 cm^3)				
0.25	16.3	16.0	10.5	d too large
0.5	11.5	11.0	6.5	
1.0	8.14	8.0	10.5	*
1.5	6.65	6.5	9.3	
2.0	5.76	5.5	6.5	
2.5	5.15	5.0	8.1	
3.0	4.70	4.5	6.6	$^t/_d$ too large
(60 $^T/_{14}$ Schübring shield, 5960 cm^3)				
$^1/_2$	3.08	3.0	8.7	d too small
$^1/_3$	3.77	3.5	5.0	$^d/_x$ too small
$^1/_4$	4.35	4.25	9.0	odd d
$^1/_5$	4.87	4.5	4.8	$^d/_x$ too small
$^1/_6$	5.33	5.25	11.2	odd d
$^1/_8$	6.16	6.0	8.7	
$^1/_{10}$	6.88	6.5	5.7	$^t/_d$ too small
$^1/_{12}$	7.54	7.5	19.1	$^d/_x$ too large
$^1/_{16}$	8.71	8.5	9.0	
$^1/_{20}$	9.74	9.5	8.9	
$^1/_{25}$	10.88	10.5	7.3	d too large

normal hoplon: unlikely). These natural limits determine the scope of Table 2. (The table *is* extended below the natural limit t = $^1/_{10}$ *d* in Schübring's case: see below.) It is to be noted that only one size (and shape) is likely for the smaller (Schübring) offering: t = $^1/_8$ *d*, d = 6 *p* (i.e., 0.23 cm by 177cm), bellied with $^d/_x$ = 8.7 (i.e., bulge x = 20.3 cm). For the larger 60 T offering several choices are possible: the best is t = 1 *d*, d = 8 *p*, close to the size arrived at by Calder (here t = 1.84 cm, d = 236 cm), but with a belly represented by $^d/_x$ = 10.4 (i.e., bulge x = 22.7 cm).[17]

The smaller shield is very thin and would have been somewhat flimsy. We may make this remark somewhat more precise by noting that the stiffness of gold (0.6) is roughly three times greater than that of lead (0.2) and three times less than that of bronze (1.8 to 2.0).[18] Gold shields

[17] For what it is worth, the bulge x of the smaller shield is ca. $^2/_3$ Ionic foot or $^4/_{15}$ the width of the second step, while the bulge x of the larger shield is ca. $^3/_4$ Ionic foot or $^3/_{10}$ the width of the second step. The ratio associated with the larger shield appears more likely, but this may not be significant as it is not clear there need have been any correlation between the step width and the shield bulge (constructed at different times). Perhaps of greater significance is that the smaller shield has a bulge of close to 11 *d*, while the larger shield has a bulge of about 12 *d* - the 12 *d* is a rounder figure than the 11 and hence more likely.

[18] The stiffness is found in the usual way by dividing the modulus of elasticity by the density. The numbers given in the text are in mixed units of Mpsi (mega pounds per square inch) per gm/cm^3

of thickness decreasing below ca. $^1/_{10}$ *d* (0.18 cm) would show an increasing tendency to sag under their own weight. Thus for shields this thin a (presumably wooden) support structure would be required, and this the inscription does not mention. Though remote, this possibility cannot be entirely ruled out as the ordinary *hoplon* frequently had a wooden frame.[19]

4. In the absence of external evidence on the size and shape of the offering, we must rely on evidence internal to the inscription. The two most likely amounts of gold are ca. 1572 kg and ca. 112 kg. Each of these has one most probable shape and size (as discussed above). Choosing between the two is difficult, but the smaller shield seems flimsier or would require an unmentioned frame, and the larger shield has slightly more "natural" dimensions. There are a number of uncertainties in our attempt to deduce the size and shape of this offering: the density of the gold used, the conversion factor from gm to mina, the conversion factor from cm to Greek feet (*p*), the inexactness of the ancient measures of weight (i.e., how close to 60 T of gold did the smith come?) and the approximation involved in modelling a hoplon as a rimless spherical cap (hopla usually had a narrow ring-shaped flat rim). None of these are large (their combined effect is probably ca. 5%) but they do suggest caution. Nevertheless the tendency is in favor of the full 60 T hoplon ("spherical cap") shield, in particular one 1 *d* (1.84 cm) in thickness and 8 *p* (236 cm) in diameter, having a curvature represented by $^d/_x = 10.4$ (i.e., bulge x = 22.7 cm or ca. 12 *d*).[20]

Appendix: Formula for Volume of a Spherical Cap

There are two possible approaches: the calculus and geometry. They yield the same result. Both are presented here for completeness, as a mutual check and because different scholars will prefer different approaches.

By the calculus, we first find the surface area of the spherical cap (see Fig. 1) and then integrate to find the volume. The surface area is found by the standard method of rotating a curve about an axis,[21] here a sector of a circle about its central axis: see Fig. 2.

(stiffness has units of L^2/T^2). Valuers are given in CRC[63] (above, n. 10), p. D190 for some metals and alloys, but for silver and gold consult *Metals Handbook*[9] (above, n. 10), pp. 671 and 680. A similar set of numbers could be constructed by using the yield stress instead of the modulus of elasticity. In either case, details of the sag will depend on alloy, shape and method of manufacture.

[19] See Snodgrass[1], 61 and 63 on the use of wooden frames. Conceivably the shield on a frame could be made quite thin—hence the downward extension of Table 2 (but below $^1/_{10}$ *d* it is doubtful whether there was sufficient control of the thickness to insist on round figures). I am indebted to Katherine J. Ware for the suggestion that a gold shield might have been made in this way. The possibility seems remote to me: from the participle ἐλά[cα]ντα[c we infer a shield, from the extraordinary weight of a full 60 T gold shield Schübring suggests a $^{60}/_{14}$ T gold shield, and finally from the thickness of such a shield we suggest a wooden frame (apparently to be implied in the participle, which now bears more weight than it can stand). It seems simpler to retain the full 60 T and relieve the participle of the burden of the frame.

[20] My attention was first drawn to this inscription and the problem of the offering by Prof. W.M. Calder III some years ago, and this paper derives from his Greek Epigraphy Seminar (1986). I am indebted to him for discussion and advice.

[21] In all the textbooks. See for example G.B. Thomas, Jr. *Calculus and Analytic Geometry* ed. alt. (Addison-Wesley 1972) 262-267.

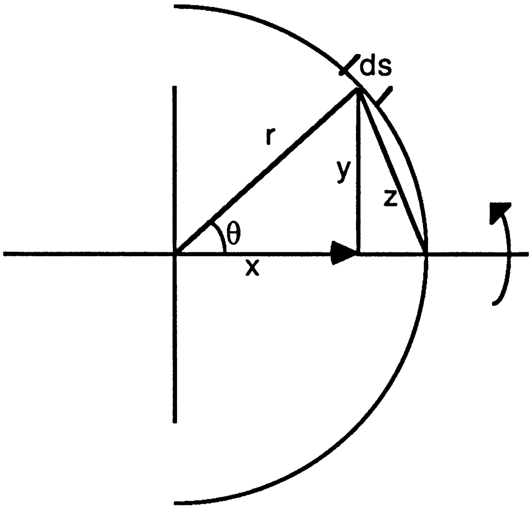

Figure 2: surface area of spherical sector by the calculus

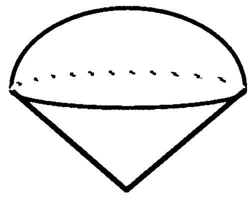

Figure 3: spherical sector

The line element is $ds^2 = dx^2 + dy^2$, and we set $x = r\cos\theta$, $y = r\sin\theta$, so that we have $ds^2 = r^2 d\theta^2$. Then the area element is $dS = 2\pi y ds = 2\pi r^2 \sin\theta\, d\theta$. Upon integration we have $S = 2\pi r^2 (1-\cos\theta)$. Now we find:

(2) $$V = \int_r^{r+t} S dr = \frac{2\pi(1-\cos.\theta)\ [(r+t)^3 - r^3]}{3}$$

This may be simplified in the case that t is small compared to r:

(3) $$V = 2\pi(1-\cos\theta)\, r^2 t$$

The geometrical approach is found in Archimedes, *On the Sphere and Cylinder* I, who gives (prop. 42) for the surface of a spherical segment (as in Fig. 3), the circle whose radius is equal to the straight line from the vertex to the circumference on the base (the line z in Fig. 2), and for the volume of the same, prop. 44, the cone with a base of area equal to the surface of the segment (as above) and a height equal to the radius of the sphere.[22] Then the volume of our spherical cap is found by subtracting the volume of the outer spherical sector (r+t) from the inner (r).

The volume is thus:

(4) $$V = \frac{A_o (r+t) - A_i r}{3}$$

[22] See T.L. Heath, *The Works of Achimedes* (Cambridge 1897) 52-55 or E.J. Dijksterhuis, *Archimedes* (Copenhagen 1956) 185-7. In any good textbook of solid geometry there is a discussion of spherical sectors. See for example G. Wentworth and D.E. Smith, *Solid Geometry* (Boston/New York/Chicago/London 1913) §§ 702, 708.

where Ao and Ai are the areas of the bases of the outer and inner sectors. These areas are the areas of the zone (of one base) of the sphere (cp. the [ant]arctic zone of the Earth), which area is (for the inner case):

(5) $$A = 2\pi rx = 2\pi r^2(1-\cos\theta)$$

(Wentworth-Smith, §§ 683 and 691) where $x = r(1-\cos\theta)$ is the height of the zone (cp. Fig. 1), and we would have (r+t) for r in the outer case. Thus our volume is:

(6) $$V = \frac{2\pi\ (1-\cos\theta)\ [(r+t)^3 - r^3]}{3}$$

evidently the same as that found by the calculus (above, formula 2).

The case of t small compared to r (formula 3) is the most likely case (for a shield) and the one we will restrict ourselves to. The dimensions d (diameter) and x (height of shield center above rim) are more accessible than θ or r (cp. Fig. 1), so we express the volume V in terms of these (and t). We have (from Fig.1) $r^2 = (d/2)^2 + (r-x)^2$ which gives $r = \frac{[x^2 + (d/2)^2]}{2x}$. The term $r(1-\cos\theta)$ in formula (3) equals x, which leaves one term r to be expressed in terms of x and d as just found.

Then we have (after some straightforward algebra):

(7) $$V = \frac{\pi t\ (4x^2 + d^2)}{4}$$

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