Regulatory Treatment of the Double Default Effect under the New Basel Accord: How Conservative is It?

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Abstract:

Within the Internal Ratings-based Approach of the New Basel Accord, banks have the possibility to consider the so-called double default effect of guaranteed exposures. However, the correlation assumptions inherent in the regulatory recognition of the double default effect appear to be quite conservative. To evaluate the degree of conservatism, on one hand, the regulatory correlation assumptions are compared with the results of a broad range of empirical studies. On the other hand, additional simulation experiments are carried out. While the comparison with the empirical results indeed suggests that the correlation parameters assumed by the regulatory authorities are much too large, the simulation experiments show that the assumed values are not unrealistic for capturing the intended effects.

Keywords: asset return correlation, concentration risk, double default effect, New Basel Accord, wrong-way risk

JEL classification: G 21, G 28

1 Introduction

In June 2005 the Basel Committee on Banking Supervision (BCBS) released an supplementary document, which, among others, contained the regulatory treatment of the double default effect (see BCBS (2005)).[1] If the credit risk of an exposure is hedged by a guarantee or a credit derivative, a credit loss is only possible when both, the obligor and the provider of the credit risk protection, default simultaneously. This simple observation is called the double default effect. In the revised framework of the New Basel Accord (see BCBS (2004)), this kind of credit risk mitigation technique was only insufficiently recognized by the substitution approach. Under this approach, a bank can substitute the risk weight of the obligor by that one of the guarantor. The maximum capital relief which can be achieved by hedging an exposure within the substitution approach is equal to the reduction in the capital requirement which can be obtained by replacing the exposure to the original obligor by a direct exposure to the protection seller. Meanwhile, banks applying the Internal Ratings-based (IRB) approach have the choice between using the substitution approach and the double default framework outlined in the following. This choice can be different for different exposures. For the standardized approach to credit risk, still only the substitution approach is available to recognize the credit risk mitigation effect of guarantees or credit derivatives.

As it will become apparent in the following, the double default framework represents a much more risk-sensitive method for recognizing the risk mitigating effect of credit risk protections as the substitution approach formerly favored by the BCBS. However, even this revised treatment of guarantees and credit derivatives contains many conservative assumptions, in particular with respect to the stochastic dependence between obligors and guarantors and within the group of guarantors. The purpose of this paper is to evaluate the degree of conser-

[1] Meanwhile, the double default framework is also part of the second draft of the German 'Solvabilitätsverordnung' (SolvV) which transforms large parts of the New Basel Accord into German law (see Solvabilitätsverordnung (2006, § 86 Abs. 3)).

vatism on which the double default framework is based. For this, on one hand, a meta study is carried out, in which existing empirical literature on credit risk dependencies is analyzed and compared with what is assumed for the regulatory treatment of the double default effect. On the other hand, simulation experiments on contagion effects and (name) concentration risk are done to further evaluate the degree of conservatism inherent in the correlation assumptions of the BCBS.

The paper is structured as follows. In section 2, the regulatory treatment of the double default effect for hedged exposures under the New Basel Accord is outlined. In section 3, methods and results of empirical studies analyzing the asset return correlations of obligors are reviewed. In section 4, additional tests are carried out for evaluating specific correlation assumptions. Finally, in section 5, the main results are summarized.

2 Regulatory Treatment of the Double Default Effect for Hedged Exposures under the New Basel Accord

In contrast to the simple substitution approach, the additional regulatory treatment of hedged exposures in the banking or trading book, proposed by BCBS (2005) for the IRB-approach, basically recognizes that the joint default probability of a hedged exposure is usually much smaller than the minimum of the individual default probabilities of the obligor (o) and the guarantor (g).[2] For this, the same modeling framework as already used for deriving the IRB-risk-weight-function, the asymptotic single-risk factor framework (ASRF), is employed (see for the following Heitfield (2003), the appendix in Heitfield and Barger (2003), and BCBS (2005)).

[2] A ‘double recovery effect’, which means that if both the obligor and the guarantor default, the bank might be able to pursue recoveries from both counterparties, is neither considered in the standardized approach nor in the IRB-approach.

It is assumed that the asset return $R_{o(g)}$ of each obligor o (guarantor g) can be represented as the sum of one systematic risk factor Z and one firm-specific risk factor $\varepsilon'_{o(g)}$:

$$R_{o(g)} = \sqrt{\rho_{o(g)}} Z + \sqrt{1-\rho_{o(g)}} \varepsilon'_{o(g)} \tag{1}$$

where $Z, \varepsilon'_o, \varepsilon'_g \sim N(0,1)$ and $Cov(Z, \varepsilon'_o) = Cov(Z, \varepsilon'_g) = Cov(\varepsilon'_o, \varepsilon'_g) = 0$. The default of an obligor o (guarantor g) is modelled as an insufficient asset return, which is below some critical level $c_{o(g)}$ at the risk horizon of one year:

$$\{\text{default of obligor } o \;\; (\text{guarantor } g)\} \Leftrightarrow \{R_{o(g)} < c_{o(g)}\}. \tag{2}$$

As $R_{o(g)} \sim N(0,1)$, the default threshold is $c_{o(g)} = \Phi^{-1}(PD_{o(g)})$ where $PD_{o(g)}$ is the unconditional one-year default probability of obligor o (guarantor g) and $\Phi^{-1}(\cdot)$ the inverse of the cumulative density function of the standard normal distribution. For deriving the IRB-risk-weight-function, it is assumed that, conditional on a realization of the systematic credit risk factor Z, the asset returns of all obligors and, hence, the default events are stochastically independent. However, for the regulatory treatment of the double default effect for hedged exposures, the BCBS assumes that an obligor and its guarantor are more closely related than only via their common dependence on the systematic risk factor Z. Thus, a further risk factor $X_{o,g} \sim N(0,1)$ is introduced which only affects the asset returns of the obligor o and the guarantor g, but not the asset return of any other obligor $o^* \neq o$ in the bank's portfolio:

$$R_{o(g)} = \sqrt{\rho_{o(g)}} Z + \sqrt{1-\rho_{o(g)}} (\sqrt{\psi_{o,g}} X_{o,g} + \sqrt{1-\psi_{o,g}} \varepsilon_{o(g)}) \tag{3}$$

where all random variables are standard normally distributed and stochastically independent. Thus, the asset return correlation between two obligors o and o^* and between two guarantors g and g^*, respectively, equals:

$$Corr(R_o, R_{o^*}) = \sqrt{\rho_o \rho_{o^*}}, \; Corr(R_g, R_{g^*}) = \sqrt{\rho_g \rho_{g^*}}, \tag{4}$$

and the correlation $\rho_{o,g}$ between the asset returns of obligor o and the guarantor g is given by:

$$\rho_{o,g} \equiv Corr\left(R_o, R_g\right) = \sqrt{\rho_o \rho_g} + \sqrt{(1-\rho_o)(1-\rho_g)}\psi_{o,g} \tag{5}$$

Within this model, the joint default probability of obligor o and guarantor g is:

$$\begin{aligned} & P\left(\{\text{default of obligor } o\} \cap \{\text{default of guarantor } g\}\right) \\ & = P\left(R_o < c_o, R_g < c_g\right) \\ & = \Phi_2\left(c_o, c_g, \rho_{o,g}\right) \\ & = \Phi_2\left(\Phi^{-1}(PD_o), \Phi^{-1}(PD_g), \rho_{o,g}\right) \end{aligned} \tag{6}$$

where $\Phi_2\left(\cdot,\cdot,\rho_{o,g}\right)$ is the cumulative density function of two normally distributed random variables with mean zero, variance one and correlation $\rho_{o,g}$.

Given the above simplified credit portfolio model with one systematic risk factor and given the additional assumption of infinite granularity of exposures,[3] Gordy (2003) has shown (for $\psi_{o,g} = 0$) that, with a probability of q, the bank does not default within the risk horizon of one year due to losses in their portfolio consisting of credits with a remaining time to maturity of one year when it holds for each unit of exposure at default EAD capital equal to the exposure's conditional expected loss $E\left[1_{\{R_o < c_o\}} LGD_o \middle| Z = Z_{1-q}\right]$. Here, Z_{1-q} is the $(1-q)$-percentile of the systematic risk factor Z, and LGD_o is the loss given default associated with an exposure to obligor o. Assuming that the loss given default is non-stochastic, the conditional expected loss function for an unhedged exposure per unit EAD equals:

[3] Note that the only systematic risk factor is still Z because the additional factor $X_{o,g}$ is specific to a pair of obligor and guarantor, but does not have any influence on the asset returns of all other obligors in the bank's portfolio.

$$E\left[1_{\{R_o<c_o\}}LGD_o\middle|Z\right]=E\left[1_{\{R_o<c_o\}}\middle|Z\right]LGD_o=P\left(R_o<c_o\middle|Z\right)LGD_o$$
$$=\Phi\left(\frac{\Phi^{-1}(PD_o)-\sqrt{\rho_o}Z}{\sqrt{1-\rho_o}}\right)LGD_o. \tag{7}$$

Plugging into (7) $Z=Z_{0.001}=\Phi^{-1}(0.001)$, subtracting PD_oLGD_o and multiplying all terms with 12.5 and 1.06, this is the IRB-risk-weight-function for *unhedged* exposures with a remaining time to maturity of one year. For a hedged exposure, the conditional expected loss function becomes:

$$E\left[1_{\{R_o<c_o\}}1_{\{R_g<c_g\}}LGD_oLGD_g\middle|Z\right]=P\left(R_o<c_o,R_g<c_g\middle|Z\right)LGD_oLGD_g$$

$$=P\left(\sqrt{\rho_o}Z+\sqrt{1-\rho_o}(\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_o)<c_o,\sqrt{\rho_g}Z+\sqrt{1-\rho_g}(\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_g)<c_g\middle|Z\right)$$
$$\cdot LGD_oLGD_g$$
$$=P\left(\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_o<\frac{c_o-\sqrt{\rho_o}Z}{\sqrt{1-\rho_o}},\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_g<\frac{c_g-\sqrt{\rho_g}Z}{\sqrt{1-\rho_g}}\middle|Z\right)$$
$$\cdot LGD_oLGD_g$$

$$=\Phi_2\left(\frac{\Phi^{-1}(PD_o)-\sqrt{\rho_o}Z}{\sqrt{1-\rho_o}},\frac{\Phi^{-1}(PD_g)-\sqrt{\rho_g}Z}{\sqrt{1-\rho_g}},\psi_{o,g}\right)LGD_oLGD_g$$

$$\underset{(5)}{=}\Phi_2\left(\frac{\Phi^{-1}(PD_o)-\sqrt{\rho_o}Z}{\sqrt{1-\rho_o}},\frac{\Phi^{-1}(PD_g)-\sqrt{\rho_g}Z}{\sqrt{1-\rho_g}},\frac{\rho_{o,g}-\sqrt{\rho_o\rho_g}}{\sqrt{(1-\rho_o)(1-\rho_g)}}\right)LGD_oLGD_g \tag{8}$$

where the second last equality follows from the fact that $\left(\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_{o(g)}\right)\sim N(0,1)$

and

$$Corr\left(\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_o,\sqrt{\psi_{o,g}}X_{o,g}+\sqrt{1-\psi_{o,g}}\varepsilon_g\right)=\psi_{o,g} \tag{9}$$

due to the independence assumptions above. Now, plugging into (8) $Z=\Phi^{-1}(0.001)$, subtracting $\Phi_2\left(\Phi^{-1}(PD_o),\Phi^{-1}(PD_g),\rho_{o,g}\right)LGD_oLGD_g$ and multiplying all terms with 12.5 and 1.06 would yield the appropriate IRB-risk-weight-function for a *hedged* exposure with a remaining time to maturity of one year within the IRB-modelling framework.[4] The substitution approach for the IRB-methods corresponds to setting in (8) $\psi_{o,g}=1$, $\rho_o=\rho_g$, and $LGD=100\%$ for that counterparty with the larger PD (see Heitfield and Barger (2003, p. 36)).

Instead of employing the exact conditional expected loss function (8) as the base for computing the risk weight for hedged exposures, the Basel Committee has proposed the following simplified approach for considering the double default effect (see BCBS (2005, 225)). The capital requirement K_{DD} per unit EAD for a hedged exposure is computed by multiplying the corresponding capital requirement K_0 for an unhedged exposure with an adjustment factor which only depends on the default probability PD_g of the guarantor:

$$K_{DD}=K_0\cdot(0.15+160\cdot PD_g)\,. \tag{10}$$

The calculation of K_0 is based on the usual risk-weight function for corporate exposures,[5] but with LGD_o replaced by the guarantor's LGD_g and with $\min\{PD_o,PD_g\}$ employed for the maturity adjustment coefficient $b(PD)$:

$$\begin{aligned}K_0=1.06\cdot LGD_g\cdot&\left(\Phi\left(\frac{\Phi^{-1}(PD_o)+\sqrt{\rho_o}\Phi^{-1}(0.999)}{\sqrt{1-\rho_o}}\right)-PD_o\right)\\&\cdot\frac{1+(M-2.5)\cdot b(\min\{PD_o,PD_g\})}{1-1.5\cdot b(\min\{PD_o,PD_g\})}\end{aligned} \tag{11}$$

The adjustment factor $(0.15+160\cdot PD_g)$ represents a linear approximation of the ratio of the capital requirement for a hedged exposure resulting from an application of the exact condi-

4 As the Basel Committee expects banks not to make provisions for the hedged part of an exposure and as the expected loss $\Phi_2\left(\Phi^{-1}(PD_o),\Phi^{-1}(PD_g),\rho_{o,g}\right)LGD_oLGD_g$ would be rather small, this term is set equal to zero by the Basel Committee (see BCBS (2005, 230)). Thus, regulatory capital is also needed for the expected loss of a hedged exposure. Furthermore, as the 'double recovery effect' is not recognized, the setting $LGD_o=1$ has to be used.

5 The risk-weight function for corporate exposures has also to be used for hedged exposures to small and medium firms that qualify as retail exposures.

tional expected loss function (8) and the capital requirement for the unhedged exposure according to the usual IRB-formula (see figure 1 and Schulte-Mattler (2006, p. 58)). For the former, the asset return correlation ρ_o of the obligor is computed according to the usual PD-dependent asset return correlation formula provided in the revised framework (see BCBS (2004, 272)). The two other correlation parameters in (5) are assumed to be:

$$\rho_g = 0.7 \text{ and } \rho_{o,g} = 0.5. \quad (12)$$

According to the Basel Committee, these parameter values correspond to the median values observed in empirical studies, which, however, are not described in more detail. These values are much larger than the maximum correlation value of 24% in the IRB-approach. As a consequence, the capital requirement which results from (10) is not always smaller than the capital requirement which results from an application of the simple substitution approach. Hence, on one hand, it has been argued, in particular by the banking industry, that the above correlation assumptions are too conservative. On the other hand, the Basel Committee has argued that these large correlation values are intended to capture specific effects. In particular, the large value for ρ_g should take into account that the group of potential guarantors is highly concentrated.[6] For example, Heitfield and Barger (2003, pp. 27) claim that the market for credit derivatives is dominated by a dozen global commercial and investment banks. The increase in concentration risk is one of the main concerns of the Basel Committee with respect to the regulatory recognition of the double default effect and resulting capital reliefs. The large value for $\rho_{o,g}$ is intended to account for 'wrong-way-risk', in particular, a simultaneous increase in the default probabilities of the obligor and the guarantor. Possible reasons for

6 Furthermore, it could be argued that credit protection might be sold mainly by large, well-diversified financial institutions. These firms are expected to have a lower default probability, but exhibit a larger exposure to systematic risk than average corporate obligors (see Heitfield and Barger (2003, p. 12, p. 15)). Similarly, the BCBS assumes within the IRB-approach that the asset return correlation is increasing with rising firm size (for firms with a yearly turn over between 5 and 50 million EUR) and is higher for large firms than for small and medium firms. This assumption is, more or less, supported by the empirical findings of Düllmann and Scheule (2003) and Lopez (2004), and is also used by the credit portfolio model CreditMetrics for estimating the firms' sensitivity to systematic risk factors (see Hahnenstein (2004, pp. 364)).

‘wrong-way-risk’ might be that protection sellers specialize in bearing risks in particular industries or regions, or that they have financial dealings with the obligors whose risk they guarantee (see Heitfield and Barger (2003, p. 19)).[7] The BCBS stresses that one of the operational requirements for the recognition of the double default effect is that there is no “excessive correlation” between credit quality changes of the obligor and the guarantor due to a common dependence on risk factors other than the systematic risk factor of the Basel II-One-Factor-Model (see BCBS (2005, 216). Due to the controversial standpoints of the banking industry and regulatory authorities, the purpose of this paper is to evaluate how conservative the correlation assumptions are really.

- insert figure 1 about here -

The Basel Committee has defined a specific scope of application and operational requirements with respect to the protection provider, the obligors and the form of protection which all have to be fulfilled before the regulatory treatment of the double default effect described above can be applied. Protection providers must be financial firms (banks, investment firms, or insurance companies), which have an appropriate expertise in the area of credit risk protection provision, are of high credit quality and possess a sufficient transparency. With respect to the obligors, only corporate exposures, exposures to small or medium firms that qualify for the retail portfolio, and claims on a public sector entity that is not a sovereign exposure are eligible. In particular, exposures to financial firms and firms belonging to the same group as the protection provider are excluded. Furthermore, the bank must ensure that their risk management systems can detect an excessive stochastic dependence between the credit quality of the obligor and the corresponding protection provider. The only forms of protection which are recognized are guarantees and credit derivatives (single-name and basket products) fulfill-

7 Heitfield and Barger (2003, p. 24) mention as an example of a manifestation of wrong-way-risk the Russian debt crisis, where U.S. banks hedged their credit exposure to Russian firms with credit protections sold by Russian banks. However, as the credit quality of the Russian obligors declined, also the credit quality of the Russian banks deteriorated.

ing the minimum operational requirements outlined already in the revised framework as well as some additional requirements (see BCBS (2005, p. 52)).

3 Empirical Evidence on the Stochastic Dependence between Credit Quality Changes

Empirical papers about the stochastic dependence between credit quality changes can be categorized along several lines. First, it can be differed between papers which estimate default correlations and those which estimate asset return correlations. Default correlations, which are defined as the correlation of two indicator functions indicating the default of two entities n and m, and asset return correlations as defined within the Basel II-One-Factor-Model are closely related by:

$$\rho^{\text{default}} = \frac{\Phi(\Phi^{-1}(PD_n),\Phi^{-1}(PD_m),\rho^{\text{asset return}}) - PD_n PD_m}{\sqrt{PD_n(1-PD_n)PD_m(1-PD_m)}}. \tag{13}$$

Based on rating- and industry-specific default frequencies of corporate bonds provided by rating agencies, Lucas (1995), Nagpal and Bahar (2001), and De Servigny and Renault (2003) estimate default correlations. These papers mainly differ in the non-parametric way how joined default probabilities are computed.[8] However, as the employed marginal default probabilities are not always explicitly reported, the estimated default correlations cannot easily be transformed into the corresponding asset return correlations. Thus, a direct comparison with the results of other empirical studies on the estimation of asset return correlations is difficult.

Second, papers can be differentiated according to the employed data. Mainly, there are, on one hand, approaches which use default data, and, on the other hand, paper which rely on data about equity or asset returns.

[8] For an overview, see Akhavein et al. (2005).

Third, the employed estimation technique is a differentiating characteristic. Papers using default data for estimating asset return correlations mainly employ either the method of moments or a maximum likelihood (ML) approach, both frequently based on the Basel II-One-Factor-Model. These two parametric approaches are shortly sketched in the following sections 3.1 and 3.2. In contrast, papers relying on equity (asset) returns often estimate a linear factor structure for explaining these returns and derive afterwards the asset return correlations from the correlations of the factors and the factor sensitivities.

Finally, a fourth, rather small, group of papers deals with the more fundamental question how the stochastic dependence between credit quality changes, beyond simple correlations, really looks like and, in particular, what the correct copula function is for describing this dependence. Some authors just demonstrate that the economic capital of a credit portfolio heavily depends on the distributional assumption for the asset returns. For example, changing the joint distribution of the asset returns from a multivariate normal to a multivariate t-distribution causes fatter tails of the portfolio loss distribution (see, e.g., Frey et al. (2001)). In other papers, it is tried to validate these differing distributional assumptions empirically. For example, Mashal et al. (2003a, 2003b) provide empirical evidence that a t-copula with 12 degrees of freedom reflects the so-called tail dependence[9] of asset returns better than a Gaussian copula, which is tail *in*dependent.

3.1 ML-Estimation Method Based on the Basel II-One-Factor-Model[10]

For ML-estimations of asset return correlations based on the Basel II-One-Factor-Model, the presentation of the modeling framework (see section 2) has to be extended by a time index for

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9 Intuitively, the coefficient of tail dependence makes a statement about the presence of joint extreme events; for a formal definition see, e.g., Schönbucher (2003, p. 332).

10 For a discussion of the pros and cons of maximum likelihood and method of moments estimation of asset return correlations, see also Gordy and Heitfield (2002).

all random variables. Thus, the normalized return $R_{n,t}$ on firm n's assets at time t is assumed to be explained by:

$$R_{n,t} = \sqrt{\rho} Z_t + \sqrt{1-\rho}\varepsilon_{n,t} \tag{14}$$

where $Z_t \sim N(0,1)$, $\varepsilon_{n,t} \sim N(0,1)$ ($n \in \{1,\ldots,N_t\}$) are standard normally distributed and $Cov(Z_t, \varepsilon_{n,t}) = Cov(\varepsilon_{n,t}, \varepsilon_{m,t}) = 0$ ($n,m \in \{1,\ldots,N_t\}$, $n \neq m$, $t \in \{1,\ldots,T\}$). The asset return correlation between any two obligors is ρ. All random variables are assumed to be serially independent. An obligor n defaults at time t if his asset return at this time falls short of some threshold c_n:[11]

$$\{\text{default of obligor } n\} \Leftrightarrow \{R_{n,t} < c_n\}. \tag{15}$$

Considering (14), the probability for the event on the right-hand side of (15), the conditional default probability $PD_{n,t}(Z)$ of obligor n at time t, equals:

$$PD_{n,t}(Z) = \Phi\left(\frac{c_n - \sqrt{\rho} Z_t}{\sqrt{1-\rho}}\right). \tag{16}$$

Conditional on the realization of the systematic risk factor Z_t, all obligors are independent and, given $PD_{n,t}(Z) = PD_t(Z)$ and $c_n = c$ for all $n \in \{1,\ldots,N_t\}$ in a homogeneous obligor bucket, the number D_t of defaults at time t is binomially distributed with parameters N_t and $PD_t(Z)$. Furthermore, we have $E\left[PD_t(Z)\right] = PD_t$ where PD_t is the unconditional default probability of the obligors at time t, which is given, for example, by the default probability of the corresponding rating grade of the obligors. To get the unconditional binomial distribution of defaults at time t, integrating over the systematic risk factor Z is necessary. Thus, given an observed time series of defaults D_t in a homogeneous risk bucket with N_t obligors at time

[11] To allow for comparability of the estimated asset correlations with those stipulated by the regulatory authorities, time-dependent default tresholds $c_{n,t}$ and time-dependent default probabilities $PD_{n,t}$ are not considered here. As there are additional risk factors for explaining the fluctuations of the default rates in these dynamic models, the sensitivity $\sqrt{\rho}$ to the (unobservable) systematic risk factor Z decreases c.p., compared to the estimation results which the static version of the model yields (see, e.g., Hamerle et al. (2003)).

$t \in \{1,\ldots,T\}$, the unknown parameters ρ and c can be simultaneously estimated by the following ML-approach:[12]

$$(\hat{\rho},\hat{c}) = \arg\min_{\rho,c} \sum_{t=1}^{T} \ln\left(\int_{-\infty}^{\infty} \binom{N_t}{D_t} PD_t(Z)^{D_t} (1-PD_t(Z))^{N_t-D_t} \phi(z)dz \right) \tag{17}$$

where $\phi(\cdot)$ is the density function of the standard normal distribution.

3.2 Method of Moments Based on the Basel II-One-Factor-Model

The method of moments employs the fact that the variance of the conditional default probability $PD(Z)$ of a homogeneous risk bucket in (16) equals[13]

$$Var\left(PD(Z)\right) = \Phi_2\left(\Phi^{-1}(PD), \Phi^{-1}(PD), \rho\right) - PD^2 \tag{18}$$

in the Basel II-One-Factor-Model. Substituting PD by the sample mean $\overline{p}$ of the time series of realized default rates $PD_1,\ldots,PD_T$ of the respective risk bucket,

$$\overline{p} = \frac{1}{T}\sum_{t=1}^{T} PD_t \,, \tag{19}$$

and $Var\left(PD(Z)\right)$ by the sample variance s^2,[14]

$$s^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(PD_t - \overline{p}\right)^2 \,, \tag{20}$$

the asset return correlation ρ can be computed from (18). Thus, in the method of moments approach, the estimation of the asset return correlations is essentially based on the volatility of the historical default rates.

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12 If the asset return correlations ρ_b and default thresholds c_b are simultaneously estimated for all buckets $b \in \{1,\ldots,B\}$, they are called 'full information maximum likelihood estimators' (see Gordy and Heitfield (2002), Düllmann and Scheule (2003)). In contrast to (17), these estimators are asymptotically efficient.

13 See Gordy (2000, p. 133), Bluhm et al. (2003, p. 119), Düllmann and Scheule (2003, p. 9).

14 Gordy (2000, p. 146) proposes a modified sample variance estimator which takes into account the finite number of obligors in the sample from which the default rates are computed. This estimator is also used by Dietsch and Petey (2002).

3.3 Empirical Results and Comparison with the Correlation Assumptions Inherent in the Regulatory Treatment of the Double Default Effect

Table 1 summarizes the range of asset return correlations observed in various empirical studies as well as the data and methodology employed for these studies. Obviously, the estimated asset return correlations are usually much smaller than those ones stipulated by the Basel Committee, in particular when default rate data is used. However, employing equity data directly or indirectly, larger asset return correlation parameters can also be observed. For example, Hahnenstein's One-Index-Model approach, which is directly based on equity returns and is most closely related to the approach proposed in the technical document of CreditMetrics (see Gupton et al. (1997)), can produce a wider range $\hat{\rho} \in [-56.8\%, 84.7\%]$ of correlation estimates (see Hahnenstein (2004)).[15] The asset return correlation estimates of Lopez (2004), which are up to 45%, are also remarkably larger than those of most other studies. In his study, equity returns are indirectly used because these are transformed into asset returns by the same option-theoretic approach that is also employed by Moody's KMV for computing the Expected Default Frequency (see Saunders and Allen (2002, pp. 49)).

- insert table 1 about here -

However, even with these occasionally larger estimates, an intra-sector asset return correlation of 70%, which is assumed for the group of guarantors by the Basel Committee, appears to be pretty large. This is also true when the assumed intra-sector correlation of 70% is com-

[15] However, the results Zeng and Zhang (2002) raise some doubt that equity correlations are good proxies for asset return correlations in credit risk calculations. These authors compare the correlation of equity returns with that one of asset returns implied out of the equity returns and liability structure information by an option-pricing approach, which is also used by Moody's KMV for computing the EDF default probability measure. Zeng and Zhang (2002) find that the correlations of asset returns computed this way tend to be larger than equity return correlations for financials, utilities and low credit quality firms. In contrast to the approach of CreditMetrics, the commercial Global Correlation Model, which underlies Moody's KMV's credit portfolio model Portfolio Manager, explains asset returns (instead of equity returns) by common risk factors. Having estimated a factor model, the asset return correlations can be computed via the factor's variances and covariances and the asset returns' sensitivity to the factors. For more details, see Zeng and Zhang (2001) and Crosbie (2005).

pared with the empirical results for the financial services sector, out of which guarantors have to be in order to ensure that the double default effect is recognized by the regulatory authorities: Hamerle et al. (2002) find an asset return correlation of only 0.5% for the banking and finance sector, whereas Zeng and Zhang (2002) and Demey et al. (2004), respectively, find an asset return correlation of 33.7% and 16.1%, respectively, for the sector of financial institutions. Therefore, in section 4.2, it is analyzed to which extent an increased asset return correlation parameter might be used to compensate for the assumed high concentration in the sector of credit derivatives dealers.

Most of the studies mentioned in table 1 analyze intra-sector asset return correlations where sectors can be rating classes, industries or countries. Only a small number of studies explicitly analyze inter-sector correlations. One of these exceptions is Demey et al. (2004) who find that the average inter-sector asset return correlation is 6.1%, which is smaller than the smallest of the intra-sector correlations they find. Hrvatin and Neugebauer (2004) also find that inter-sector correlations are generally smaller than intra-sector correlations. Furthermore, they observe that banking and finance is the industry which is on average most positively correlated with other industries. Depending on the region, the average inter-industry asset return correlation of the banking and finance industry ranges from 17.5% to 26.6%. Fu et al. (2004) report average inter-banking (finance) asset return correlations of 14% (13%). Compared to these values, the assumed correlation $\rho_{o,g} = 50\%$ between an obligor's asset return and that one of a guaranteeing financial institution also seems to be rather large. That is why in section 4.1 it is analyzed how large contagion effects specific to these two counterparties must be to explain the assumed large inter-sector correlation $\rho_{o,g} = 50\%$.

Dietsch and Petey (2004) mention several reasons why the asset correlation estimated in empirical studies might be too low. First, the length of the time series employed could be to short to cover at least one complete business cycle. As a consequence, the realized yearly default rates might appear too stable and, hence, the asset return correlation too small. Second, the employed sample might be too large and, hence, quasi-exhaustive for a given economy. When banks' portfolios are smaller and less diversified than the sample, it can be expected that larger asset return correlations are observed in typical banking books. Furthermore, reported asset return correlation estimates are always average values. Thus, it cannot be excluded that there also exist larger extreme values of asset return correlations in portfolios of limited size about which the regulatory authorities should have to worry. For example, Dietsch and Petey (2004) find that the dispersion of asset return correlation estimates is larger for large firms (yearly turnover larger than €40 million) than for smaller SMEs.[16]

Some of these studies (see, e.g., Nagpal and Bahar (2001), Bluhm et al. (2003), Düllmann and Scheule (2003), Dietsch and Petey (2002, 2004), Lopez (2004)) also address the question which relationship exists between asset return correlations (default correlations), the credit quality of the obligors, and the firm size. The BCBS employs the assumption that the asset return correlation is increasing with decreasing default probability and (for small and medium firms) with rising firm size. However, the empirical results are not unambiguous and are partially controversial to the regulatory assumptions.

Finally, to show that it is real worth to think about the degree of conservatism inherent in the regulatory treatment of the double default effect, figure 2 compares the capital requirements for a hedged exposure (based on the exact conditional loss function (8)) for two different sets of correlation parameters. On one hand, the asset return correlations $\rho_g = 0.7$, $\rho_{o,g} = 0.5$, and ρ_o chosen according to the IRB-correlation formula, which are prescribed for the regulatory

[16] However, even the highest asset return correlation estimated by Dietsch and Petey (2004) for simulated portfolios of these larger firms is only 12%.

recognition of the double default effect, are employed. On the other hand, all parameters ρ_o, ρ_g, $\rho_{o,g}$ are chosen according to the IRB-correlation formula. Obviously, as expected, the conservative regulatory asset return correlation assumptions cause a significant increase in the necessary regulatory capital compared to the usual IRB-correlation assumptions. In the considered examples, this increase can be up to a factor of six.

– insert figure 2 about here –

4 Additional Evaluations

4.1 Evaluation of the Obligor-Guarantor Asset Return Correlation Assumption

The BCBS assumes a rather large correlation $\rho_{o,g} = 0.5$ between the specific asset return of the obligor and that one of the corresponding guarantor. As this correlation value is not backed by empirical findings about general inter-sector correlations (see section 3.3), assumed contagion effects between the creditworthiness of both counterparties must be responsible for this value.[17] This section is intended to demonstrate how large the assumed contagion effects must be in order to explain the value $\rho_{o,g} = 0.5$. For this, the Basel II-One-Factor-Model is extended to a simple contagion model. Within this modelling approach, the question is addressed how large the PD_g of the guarantor must increase due to contagion effects caused by a default of the obligor for whom he guarantees in order to explain the pairwise asset return correlation $\rho_{o,g} = 0.5$.

It is assumed that the asset returns of the obligor and the guarantor, respectively, are not modelled as in (3), implying that the random variables R_o and R_g are only correlated by their

[17] Contagion effects mean that the stochastic dependence between the creditworthiness of different obligors is stronger than only due to a common dependence on systematic risk factors, but that the default of one obligor influences the default probabilities and credit spreads of the surviving obligors with certainty. For contagion models see, e.g., *Davis* and *Lo* (2001), *Schönbucher* and *Schubert* (2001), or *Giesecke* and *Weber* (2004).

common dependence on the systematic risk factor Z and the obligor-guarantor-specific risk factor $X_{o,g}$. Instead, it is assumed that the compensation payments of the guarantor which are caused by a default of the obligor lead to a decrease of the guarantor's asset return with probability one and, hence, to an increase of his original unconditional default probability PD_g:

$$\begin{aligned} PD'_g &= 1_{\{R_o<\Phi^{-1}(PD_o)\}}(PD_g + \lambda_{o,g}PD_g) + 1_{\{R_o>\Phi^{-1}(PD_o)\}}PD_g \\ &= 1_{\{R_o<\Phi^{-1}(PD_o)\}}\lambda_{o,g}PD_g + PD_g \end{aligned} \tag{21}$$

with $\lambda_{o,g} > 0$. The indicator function $1_{\{R_o<\Phi^{-1}(PD_o)\}}$ is equal to one whenever the obligor has defaulted until the risk horizon, otherwise it is zero. Thus, the original unconditional default probability PD_g is increased by $(\lambda_{o,g}100)\%$ percent whenever the obligor defaults. The asset returns of the obligor R_o and the guarantor R_g, respectively, are modelled by (1). Thus, the joint default probability of the obligor and the guarantor within this simple contagion model is:

$$\begin{aligned} &P\left(\{\text{default of obligor } o\} \cap \{\text{default of guarantor } g\}\right) \\ &= \Phi_2\left(R_o < \Phi^{-1}(PD_o), R_g < \Phi^{-1}(PD'_g), \sqrt{\rho_o\rho_g}\right) \\ &= \int_{-\infty}^{\Phi^{-1}(PD_o)} \int_{-\infty}^{\Phi^{-1}(1_{\{r_o<\Phi^{-1}(PD_o)\}}\lambda_{o,g}PD_g+PD_g)} \phi_2\left(r_o, r_g, \sqrt{\rho_o\rho_g}\right) dr_g dr_o \\ &= \int_{-\infty}^{\Phi^{-1}(PD_o)} \int_{-\infty}^{\Phi^{-1}(\lambda_{o,g}PD_g+PD_g)} \phi_2\left(r_o, r_g, \sqrt{\rho_o\rho_g}\right) dr_g dr_o \\ &= \Phi_2\left(\Phi^{-1}(PD_o), \Phi^{-1}(\lambda_{o,g}PD_g + PD_g), \sqrt{\rho_o\rho_g}\right). \end{aligned} \tag{22}$$

Comparing (22) with (6), the question arises how large the percentage increase $\lambda_{o,g}$ of the original unconditional default probability PD_g of the guarantor caused by a default of the obligor has to be in order to ensure the equality of the joint default probability within the above contagion model and within the extended Basel II-One-Factor-Model described in section 2 where there is only a (strong) correlation between the asset returns of the obligor and the guarantor:

$$\lambda_{o,g}^{*} = \left\{ \lambda_{o,g} > 0 \middle| \Phi_2\left(\Phi^{-1}(PD_o), \Phi^{-1}(PD_g), 0.5\right) = \Phi_2\left(\Phi^{-1}(PD_o), \Phi^{-1}(\lambda_{o,g} PD_g + PD_g), \sqrt{\rho_o \rho_g}\right) \right\}. \tag{23}$$

In (23), for the guarantor, the prescribed intra-sector correlation parameter $\rho_g = 0.7$ is used, and the intra-sector correlation parameter ρ_o of the obligor is chosen according to the IRB-correlation formula, which yields $\sqrt{\rho_o \rho_g} \in [0.24, 0.41]$. Table 2 shows for various marginal default probabilities the parameter $\lambda_{o,g}$ that ensures the identity of the joint default probabilities in the usual Basel II-One-Factor Model and in the extended Basel II-One-Factor Contagion Model.

- insert table 2 about here -

As can be seen, the equivalence of the joint default probabilities in both modelling approaches is given if a default of an obligor roughly doubles the default probability of the guarantor. Comparing this result for example with the distance between the default probabilities 0.008%, 0.022%, and 0.185%, respectively, of obligors with rating Aa, A, and Baa, respectively (see Hamilton et al. (2006, p. 25)), the assumed doubling of the PD due to contagion effects between the obligor and the guarantor corresponds to much less than a rating downgrade of the guarantor. At best, the PD-increase corresponds to a downgrade by one rating notch, which at first sight does not seem to be a too conservative assumption. However, of course, whether this assumption is conservative or not depends on many factors, such as the amount of money for which the guarantor has guaranteed, the firm size of the guarantor and the amount of further income (out of a direct business relationship) which is lost for the guarantor due to the default of the obligor.

4.2 Evaluation of the Guarantor Asset Return Correlation Assumption

The Basel Committee assumes an intra-sector asset return correlation of 70% for the group of guarantors. As this large correlation is not found in empirical studies on the asset return correlation within the financial sector, it can be assumed that this parameter might be used to compensate for the presumed high (name) concentration risk in the sector of credit derivatives dealers which is not captured by the ASRF-model underlying the IRB-approach. In order to evaluate the adequacy of the intra-sector asset return correlation of 70%, two portfolios are considered in the following: the first portfolio is poorly diversified and consists of the dozen global commercial and investment banks which dominate the market for credit derivatives (see Heitfield and Barger (2003, pp. 27)). The second portfolio is assumed to be infinitely fine-grained, which is also the assumption of the IRB-approach. Then, it is asked how much larger (compared to the poorly diversified credit derivatives dealer portfolio) the asset return correlation in the infinitely fine-grained portfolio has to be in order to ensure that the percentiles of the default rates of both portfolios coincide at high confidence levels.

The probability distribution of the portfolios' default rate is – as usual – computed for a risk horizon of one year. For modelling the default behaviour of the obligors, the Basel II-One-Factor Model (see (1), (2) in section 2) is employed. Thus, a stochastic dependence between the credit quality changes of the obligors only comes from a common dependence on the systematic risk factor Z; an extra pairwise dependence is not assumed ($\psi_{o,g} = 0$). As explained in section 3.1, due to the conditional (on Z) independence of the obligors, the number of defaults D is binomially distributed with parameters N and $PD(Z)$. Here, N denotes the number of obligors in the portfolio, and $PD(Z)$ is the conditional default probability (see (16)). The unconditional binomial distribution of the number of defaults D is given by (see Vasicek (2002)):

$$P\left(D=d\right)=E\left[P\left(D=d\middle|Z=z\right)\right]=\int_{-\infty}^{\infty}\binom{N}{d}PD(z)^{d}(1-PD(z))^{N-d}\phi(z)dz\,. \tag{24}$$

Finally, the default rate $r_{\#12}$ of the portfolio of the dozen credit derivatives dealers is defined as $r_{\#12}=D/N$. For the infinitely fine-grained portfolio, the cumulative density function of the default rate r_{∞} is given by (see Vasicek (2002), Bluhm et al. (2003, p. 91)):

$$P(r_{\infty}\leq x)=\Phi\left(\frac{\Phi^{-1}(x)\sqrt{1-\rho}-\Phi^{-1}(PD)}{\sqrt{\rho}}\right)\quad(x\in[0,1]) \tag{25}$$

and the corresponding density function is:

$$f(x)=\sqrt{\frac{1-\rho}{\rho}}\exp\left(\frac{1}{2}(\Phi^{-1}(x))^{2}-\frac{1}{2\rho}\left(\Phi^{-1}(PD)-\Phi^{-1}(x)\sqrt{1-\rho}\right)^{2}\right). \tag{26}$$

Any α-quantile $q_{\alpha}(r_{\infty})$ of the random variable r_{∞} distributed according to (25) is given by (see Bluhm et al. (2003, p. 93)):

$$q_{\alpha}(r_{\infty})=\Phi\left(\frac{\Phi^{-1}(PD)+\sqrt{\rho}q_{\alpha}(Z)}{\sqrt{1-\rho}}\right) \tag{27}$$

where $Z\sim N(0,1)$. Formula (27) is also the base for the computation of the capital requirement within the IRB-approach (see (7)).

Table 3 shows those correlation parameters ρ^{*} which have to be assumed in the infinitely fine-grained portfolio in order to ensure that the percentiles of the default rates at high confidence levels $\alpha\in\{99.9\%,99.98\%\}$ coincide for both portfolios:

$$\rho^{*}=\left\{\rho\in[-1,1]\middle|q_{\alpha}(r_{\#12})=\Phi\left(\frac{\Phi^{-1}(PD)+\sqrt{\rho}q_{\alpha}(Z)}{\sqrt{1-\rho}}\right)\right\}. \tag{28}$$

As $r_{\#12}$ is a discrete random variable with realizations in $\{0,1/12,\ldots,1\}$, linear and cubic spline interpolations are employed for computing $q_{\alpha}(r_{\#12})$.[18] The asset return correlation in the poorly diversified portfolio of guarantors is set equal to the minimum and maximum possible values within the IRB-approach for corporate exposures. These values are 8% and 24%, respectively. As can be seen in table 3, the asset return correlations which are necessary for the obligors in the infinitely fine-grained portfolio to ensure the identity of the default rate percentiles are larger the better the rating of the guarantor is. For the high investment-grade rating Aa, the asset return correlation that is required in the infinitely fine-grained portfolio to produce the same default rate percentiles at high confidence levels as in the poorly diversified portfolio are roughly equal to the regulatory parameter value $\rho_g = 70\%$.[19] However, for lower credit qualities of the guarantors, the asset return correlation which is needed to compensate for concentration risk is overestimated. Thus, the regulatory assumption $\rho_g = 70\%$ can be understood as an upper bound for the asset return correlation surcharge that is necessary to capture the concentration risk within the group of guarantors.

- insert table 3 about here -

5 Conclusions

Within the Internal Ratings-based Approach of the New Basel Accord, banks have the possibility to consider the so-called double default effect of guaranteed exposures. However, the correlation assumptions inherent in the regulatory recognition of the double default effect appear to be quite conservative. To evaluate the degree of conservatism, in this paper, on one hand, the regulatory correlation assumptions are compared with the results of a broad range of

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18 However, the problem of the cubic spline interpolation is that it cannot be guaranteed any more that the distribution function is monotonously increasing as it should be.

19 The one-year default probabilities corresponding to the indicated ratings in table 3 are taken from Hamilton et al. (2006). However, for regulatory purposes, there is floor at 0.03% for all default probabilities as an additional security buffer. This means that even if the guarantor has an Aa or Aaa rating which implies smaller default probabilities, these smaller values cannot be employed for regulatory purposes.

empirical studies. On the other hand, additional simulation experiments are carried out to analyze to which extent concentration risks within the group of guarantors and contagion effects between obligors and guarantors can explain the regulatory correlation assumptions. While the results of the meta-study indeed suggest that the correlation parameters assumed by the regulatory authorities are much too large, the simulation experiments show that the assumed values are not unrealistic for capturing the intended effects.

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Tables

Table 1: Overview of Empirical Studies on the Estimation of Asset Return Correlations

Authors	Data	Estimation Method	Asset Return Correlation
New Basel Accord			Sovereigns and banks: $\rho^{reg} \in [12\%, 24\%]$; corporates: $\rho^{reg} \in [8\%, 24\%]$; retail: $\rho^{reg} \in [3\%, 16\%]$
Asset Return Correlation Based on Default Data			
Gordy (2000)	Standard & Poor's rating-specific default frequencies of corporate bonds in 1981-1997	Methods of moments based on the Basel II-One-Factor-Model	$\hat{\rho} \in [12.1\%, 35.4\%]$
Dietsch and Petey (2002)	Internal rating transition data of 224,000 French small and medium enterprises (SMEs) in 1995-1999, provided by a large French credit insurance company (Coface-SCRL)	Methods of moments based on the Basel II-One-Factor-Model	$\hat{\rho} \in [0\%, 15.8\%]$
Hamerle et al. (2002)	Industry-specific quarterly insolvency rates of West-German firms in 1980-2001	ML based on the Basel II-One-Factor-Model	$\hat{\rho} \in [0.5\%, 3.5\%]$; $\overline{\rho} = 0.9\%$; banking and insurance: $\hat{\rho} = 0.5\%$
Bluhm et al. (2003)	Moody's rating-specific default frequencies of corporate bonds in 1970-2000	Methods of moments based on the Basel II-One-Factor-Model plus linear regression to cope with zero PDs for upper investment grade bonds	$\hat{\rho} \in [10\%, 35\%]$; $\overline{\rho} = 23.5\%$
Düllmann and Scheule (2003)	Time series of default histories of 53,280 German privately-owned or corporate companies in 1991-2000, provided by the German Central Bank	Methods of moments and ML based on the Basel II-One-Factor-Model	Calibration to insolvency rates yields: $\hat{\rho} \in [0.2\%, 6.4\%]$, calibration to loan loss provisions yields: $\hat{\rho} \in [0.9\%, 14\%]$
Hamerle et al. (2003)	Industry- and country-specific time series of insolvency rates of G-7 countries with varying length (longest: Germany and UK with 20 years, shortest: Canada and Japan with 10 years from 1990 to 1999)	ML based on the Basel II-One-Factor-Model (only static version is considered here)	$\hat{\rho} < 2.3\%$
Rösch and Scheule (2003)	Annual charge-off rates filed by all US commercial banks which are used as a proxy of the default rate of a given year	ML based on the Basel II-One-Factor-Model	Residential real estate loans: $\hat{\rho} = 1.0\%$; credit card loans: $\hat{\rho} = 1.0\%$; other consumer loans: $\hat{\rho} = 0.7\%$

Table 1 continued

Demey et al. (2004), Demey and Roncalli (2004)	Standard & Poor's industry-specific default frequencies of corporate bonds in 1981-2002	ML based on a Two-Factor-Version of the Basel II-Model with one common factor, which affects all obligors in the same way, and one specific factor depending on the risk class an obligor belongs to	Estimation of asset return correlations within and across sectors of activity (intra- and inter-sector-correlations): $\hat{\rho}^{Intra} \in [6.8\%, 39.3\%]$; financial institutions: $\hat{\rho}^{Intra} = 16.1\%$; $\hat{\rho}^{Inter} = 6.1\%$ (the inter-sector-correlation is assumed to be constant across all sectors),
Dietsch and Petey (2004)	Internal rating transition data of 440,000 French SMEs in 1995-2001 provided by a large French credit insurance company (Coface-SCRL) and of 280,000 German SMEs in 1997-2001 provided by CreditReform	Methods of moments based on the Basel II-One-Factor-Model	France: $\hat{\rho} \in [0\%, 10.7\%]$; Germany: $\hat{\rho} \in [0\%, 6.5\%]$
Jobst and de Servigny (2005)	Standard & Poor's CreditPro ratings and default database (rating history from 1981 to 2003); Standard & Poor's Compustat (North America) data (the covered time period is from 1962 to 2003); monthly equity time-series of around 2,200 firms which are also covered by the CreditPro and Compustat data (time period from 1981 to 2003)	Based on rating data as well as equity default swap (EDS) data, asset return correlations are derived in two ways: 1) employing the approach of de Servigny and Renault (2003) for estimating joint default probabilities, default correlations are determined and transformed into asset return correlations based on the Basel II-One-Factor-Model, 2) application of the ML approach based on the Two-Factor-Version of the Basel II-Model proposed by Demey et al. (2004)	Based on rating data, average intra-industry asset return correlations are (depending on the estimation method) around 14% to 17%, and average inter-industry asset return correlations are around 4% to 7%; based on EDS data, the intra- and inter-industry asset return correlations increase significantly, furthermore, the EDS-based asset return correlation estimates are sensitive with respect to the chosen risk horizon and the EDS-barrier (in particular for barriers above 50%)
Asset Return Correlation Based on Equity (Asset) Return Data			
Zeng and Zhang (2002)	Data from Moody's KMV's Global Correlation Model estimation database: weekly equity and financial statement information of more than 27,000 firms in 45 countries and 61 industries from May 1988 to June 1999	Asset returns are derived from equity returns and liability structure information by using an option-pricing-approach; asset return correlations are computed from these synthetic time series of asset returns	Median for financial firms: $\rho^{Median} = 33.7\%$ (in contrast, the median of equity return correlations among these firms is only 19.9%)
Fu et al. (2004)	Moody's rating data on nearly 10,000 corporates and financial institutions over the period 1970 to 2002; data from Moody's KMV's Global Correlation Model estimation database for approximately 7,000 US firms	Asset return correlations are derived in two ways: 1) based on the direction of joint rating changes, 2) factor-model approach based on asset returns which are derived from equity returns and liability structure information by using an option-pricing-approach	Average intra-sector asset return correlation is 12% (15%) for method 1 (method 2); average inter-sector asset return correlation is 3% (13%) for method 1 (method 2); for method 2, the average intra-banking (finance) asset return correlation is 21% (15%) with minimum and maximum values of 7% (5%) and 63% (65%), respectively; the average inter-banking (finance) asset return correla-

Table 1 continued

			tion is 14% (13%) with maximum and minimum values of 6% (5%) and 43% (43%), respectively
Hahnenstein (2004)	55 weekly observations of equity returns of 241 German corporate companies (banking, insurance and financial services are excluded) from January 2001 to January 2002	One-index model where returns on industry-specific stock indices explain the return on individual stocks; the equity return correlations $\hat{\rho}^{equity}$ are derived from their sensitivity to the stock indices and the correlation of the stock indices' returns; assumption: $\hat{\rho}^{equity} \approx \hat{\rho}^{asset}$	$\hat{\rho} \in [-56.8\%, 84.7\%]$; $\overline{\rho} = 10.7\%$
Hrvatin and Neugebauer (2004)	Equity data of firms in the Dow Jones Global Universe: 6,100 firms which were grouped into 25 Fitch industries and 34 countries	Factor-model approach of Fitch's Default Vector Model (Version 2.0) is applied; asset return correlations are approximated by equity return correlations which are derived out of a factor model	Within the same region: $\hat{\rho}^{Intra} \in [18.8\%, 27.6\%]$, $\hat{\rho}^{Inter} \in [15.8\%, 22.7\%]$; on average, banking/finance is most positively correlated with other industries: depending on the region, the average inter-industry asset return correlation ranges from 17.5% to 26.6%
Pitts (2004)	Monthly asset values from the Moody's KMV data set supplied with Credit Monitor for 27 airlines from July 1997 to August 2001	'Mixed random and fixed effects model' which considers industry and size classification of obligors, based on a multi-industry extension of the Merton (1974) credit risk model	$\hat{\rho}^{Median} = 9.9\%$
Lopez (2004)	Loan portfolios consisting of obligors from US, Japan and Europe which are in the CreditMonitor database of Moody's KMV as of year-end 2000	To calibrate the asset return correlation parameter for a portfolio at the 99.9%-loss percentile, the absolute difference between the CVaR indicated by the unconstrained Portfolio Manager model of Moody's KMV and by the ASRF-constrained version of the Portfolio Manager is minimized: $\hat{\rho} = \arg\min_{\rho \in [-1,1]} \left\| \mathrm{CVaR}^{\mathrm{PM}} - \mathrm{CVaR}^{\mathrm{PM,\ constrained}}(\rho) \right\|$	$\hat{\rho} \in [10.0, 45.0\%]$
Akhavein et al. (2005)	Rating agency data on 7,886 US corporate issuers between January 1970 and December 2004 which were classified into one of Fitch's 25 industry categorizations; equity data underlying Fitch's Default Vector Model (Version 2.0)	Asset return correlations are derived in three ways: 1) employing different non-parametrical approaches for estimating joint default probabilities, default correlations are determined and transformed into asset return correlations based on the Basel II-One-Factor-Model, 2) based on the direction of joint rating changes, 3) equity-based factor-model approach (asset return correlations are approximated by equity return correlations which are derived out of a factor model)	Depending on the estimation method, the average intra-industry asset return correlation varies between 19.7% and 26.4%, and the average inter-industry asset return correlation ranges from 14.4% to 20.9% (the results of the ratings-based approach are excluded because this method yields significantly lower correlation values than any other method); on average, intra-industry asset return correlations are larger than inter-industry asset return correlations

Table 1 continued

Chernih et al. (2006)	Monthly asset values from the Moody's KMV data set supplied with Credit Monitor: 20,144 firms with between 40 and 107 months of asset returns each from March 1997 to March 2006	Asset returns are derived from equity returns and liability structure information by using an option-pricing-approach; asset return correlations are computed from these synthetic time series of asset returns	$\overline{\rho}^{Intra} = 11.1\%$; $\overline{\rho}^{Inter} = 6.3\%$
Düllmann et al. (2006)	Monthly asset values of 1,988 non-financial European firms in the time period January 1996 to February 2004 from the Moody's KMV data set supplied with Credit Monitor	Asset returns are derived from equity returns and liability structure information by using an option-pricing-approach; asset return correlations are computed from these synthetic time series of asset returns; for analyzing the time variation in asset return correlation estimates, overlapping 24-months time windows are used	overall cross-sectional median (market) asset return correlation is 10.2%; median (sector) asset return correlation is 12.3%; strong time variation in asset return correlation estimates can be observed (e.g., the median (market) asset return correlation varies between 4% and 16% in the observation period)

Table 2
Basel II-One-Factor Contagion Model

PD_o	0.1%	0.3%	0.5%	0.7%	0.9%
PD_g					
0.02%	1.3	1.1	1.1	1.1	1.2
0.10%	1.2	1.1	1.1	1.1	1.1
0.18%	1.2	1.1	1.1	1.1	1.1

PD_o	1%	3%	5%	7%	9%
PD_g					
0.02%	1.2	1.3	1.2	1.0	0.9
0.10%	1.1	1.3	1.2	1.0	0.9
0.18%	1.1	1.2	1.2	1.0	0.9

Table 2 shows how large the increase of the default probability of the guarantor due to contagion effects which result from a default of the obligor, for whom he guarantees, has to be in order to explain the prescribed pairwise asset return correlation of $\rho_{o,g} = 0.5$ between the obligor and the guarantor. Thus, the parameter values shown above are the solution of:

$$\lambda_{o,g}^{*} = \left\{ \lambda_{o,g} > 0 \middle| \Phi_2\left(\Phi^{-1}(PD_o), \Phi^{-1}(PD_g), 0.5\right) = \Phi_2\left(\Phi^{-1}(PD_o), \Phi^{-1}(\lambda_{o,g} PD_g + PD_g), \sqrt{\rho_o \rho_g}\right) \right\},$$

which ensures that the joint default probabilities in the usual Basel II-One-Factor Model and in the extended Basel II-One-Factor Contagion Model coincide. For the guarantor, the prescribed intra-sector correlation parameter $\rho_g = 0.7$ is used, and the intra-sector correlation parameter ρ_o of the obligor is chosen according to the IRB-correlation formula. The firm size is chosen as 50 million €.

Table 3
Evaluation of the Guarantor Asset Return Correlation

PD_g	0.008% (Aa)	0.022% (A)	0.185% (Baa)
α		$\rho_{\#12} = 8\%$	
99.98%	68.46%*	50.24%*	31.15%*
	64.56%**	48.91%**	37.32%**
99.9%	---*	---*	32.08%*
	---**	68.43%**	31.49%**
		$\rho_{\#12} = 24\%$	
99.98%	68.46%*	50.57%*	35.69%*
	64.56%**	49.25%**	36.65%**
99.9%	---*	---*	39.14%*
	---**	68.43%**	33.52%**

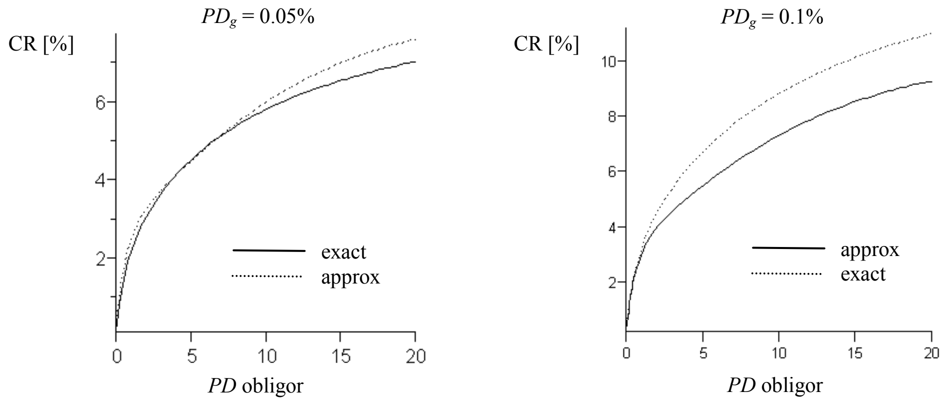
Table 3 shows those correlation parameters ρ^* which have to be assumed in the infinitely fine-grained portfolio in order to ensure that the percentiles of the default rate at high confidence levels $\alpha \in \{99.9\%, 99.98\%\}$ coincide with those of the poorly diversified portfolio of guarantors:

$$\rho^* = \left\{ \rho \in [-1,1] \middle\| q_\alpha(r_{\#12}) = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho} q_\alpha(Z)}{\sqrt{1-\rho}} \right) \right\}.$$

As $r_{\#12}$ is a discrete random variable with realizations in $\{0, 1/12, \ldots, 1\}$, linear (*) and cubic spline interpolations (**) are employed for computing $q_\alpha(r_{\#12})$. The asset return correlation in the poorly diversified portfolio of guarantors is set equal to the minimum and maximum possible values within the IRB-approach for corporate exposures. These values are 8% and 24%, respectively. If a correlation value is missing this can mean that either the 99.9%-percentile of $r_{\#12}$ cannot be computed because the probability for $r_{\#12} = 0$ is already larger than 99.9%, or that the linear interpolation of the distribution function of $r_{\#12}$ is too bad so that no asset return correlation ρ^* with $q_{99.9\%}(r_{\#12}) = q_{99.9\%}(r_\infty)$ exists. As $q_\alpha(r_\infty)$ (see (27)) is not for all default probabilities PD and confidence levels α a monotonous function in ρ, but can also be hump-shaped, the smallest possible asset return correlation fulfilling (28) is taken and displayed in table 3.

Figures

Figure 1:
Comparison of Capital Requirements for a Hedged Exposure Resulting from an Application of the Exact Conditional Expected Loss Function and the Regulatory Approximation



Parameters: $LGD_g = 45\%$, $S_o = 50$ million €, $M = 2.5$ years (no maturity mismatch).

Figure 2:
Comparison of the Exact Capital Requirements for a Hedged Exposure for Various Asset Return Correlations

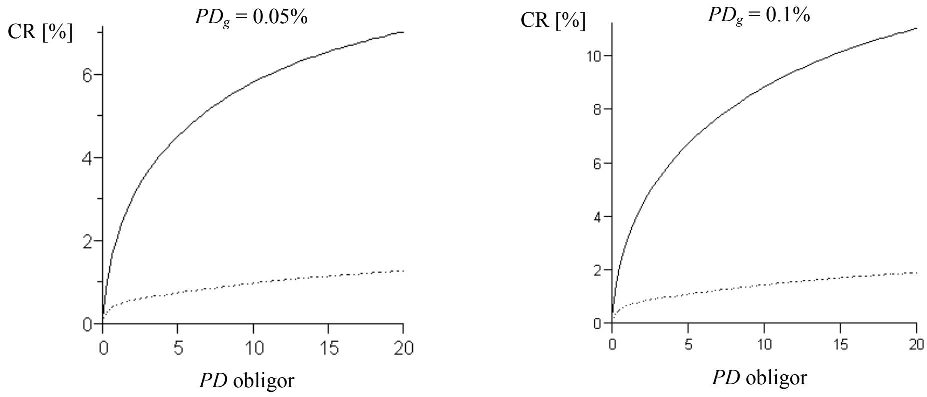


Figure 2 shows the capital requirements for a hedged exposure computed with the exact conditional expected loss function when, on one hand, the asset return correlation parameters $\rho_g = 0.7, \rho_{o,g} = 0.5$, ρ_o chosen according to the IRB-correlation formula, which are prescribed for the regulatory recognition of the double default effect, are employed (solid line) and, on the other hand, all parameters $\rho_o, \rho_g, \rho_{o,g}$ are chosen according to the IRB-correlation formula (dashed line). Parameters: $LGD_g = 45\%$, $S_o = S_g = 50$ million €, $M = 2.5$ years (no maturity mismatch).